NUS RMI Industrial Research Papers – No. 2022-01

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June 2022

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THE SCHEWEINLER-WIGNER ORTHOGONALIZATION FOR RISK MODEL CONSTRUCTION

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Abstract

In this paper, we investigate the Schweinler–Wigner/Löwdin (SW/L) orthogonalization scheme for risk model constructions. The SW/L scheme maximizes the overlaps between the original risk factors and the new orthogonalized (independent) risk factors. We test the robustness of the risk factor orthogonalization procedure on four risk models based on combinations of Farmer-French 3 factors and various sets of industry factors in both the US and the Chinese markets. We show that the original risk factors and the corresponding new orthogonalized factors are highly correlated, and such high correlations are stable over a long period. This allows the orthogonalized factors to retain the economic intuition of original explicitly-defined risk factors, leading to a better and clearer risk attribution.

1. Introduction

1.1 Systematic risk

Systematic risk, also known as “un-diversifiable risk”, refers to the risk inherent to the entire market or market segment. It affects the overall market, not just a particular asset. This type of risk is both unpredictable and impossible to avoid. Systemic risks include policy risks, macroeconomic risks, and environmental risks. Systematic risk cannot be mitigated through diversification; hedging is thus the main tool for mitigating systematic risk. To hedge the risk effectively requires a risk model for identifying risk factors and performing quantitative risk attribution.

1.2 The necessity of orthogonalization

In asset pricing and portfolio management, a multi-factor risk model is often used to explain stock returns in terms of various systematic risk factors. Under the multi-factor model framework, the systematic risk in the return of a risky asset is explained as a linear combination of the risk factors,

$$r_i(t) = \sum_k \beta_i^k f_k(t) + \epsilon_i(t),$$

where $r_i(t)$ is the return (from t-1 to t) of the asset i, $f_k(t)$ is the risk factor portfolio return, β_i^k is factor loading, and $\epsilon_i(t)$ represents the idiosyncratic part of the asset return.

If the factors are linearly independent, the factor loading determines how much systematic risk is loaded on the risk factor. Yet, the collinearity between risk factors is not normally eliminated in a risk model. This is the case for many well-known risk models, such as Barra’s risk model, which is constructed with risk factors explicitly defined. There are also risk models based on the principal component analysis (PCA); in these models, the risk factors are the principal components, and they are linearly independent. The PCA-based model, however, has a disadvantage in that the risk factors don’t have clearly defined meaning; it is difficult to perform proper hedging of such risk factors. In this paper, we study a risk model construction method using the risk factors that are both linearly independent and interpretable. We start with a set of well-defined risk factors, then employ an orthogonalization scheme to generate a new set of linearly independent factors. The crucial choice here is to use an orthogonalization method that maximizes the overlap between the new factors and the original factors we start with; a significant overlap would allow the new factors to retain the economic intuition of the original factors. Schweinler–Wigner/Löwdin (SW/L) orthogonalization method (Schweinler-Wigner, 1970; Löwdin 1950) is precisely the one we need for this purpose.

2. Orthogonalization procedure

2.1 Literature review

There have been many studies of risk factors for analyzing stock performance. The Fama-French 3-factor model (FF3, 1993) includes three well-known factors: the market, size, and value factors. Jegadeesh & Titman (1993) published the first research on the momentum factor. Chen and De Bondt(2004) further found that investors could benefit from chasing investment styles. In addition to including style factors, a good risk model also need to include factors related to market and industry exposures. In this paper, we study risk models that include both the FF3 risk factors and the factors related to sector/industry exposures.

Under a multi-factor framework, the possible covariance between the risk factors makes it complicated to attribute the sources of risk (Campbell and Mei, 1993). To eliminate the covariances among the risk factors, we need to use an orthogonalization method. The Gram- Schmidt orthogonalization method is perhaps the most used, but the Schweinler–Wigner/Löwdin method is becoming recognized in finance research. Klein and Chow (2013) use the SW/L method for factor orthogonalization in the context of examining the contribution of the Fama and French (1993) three factors to the return
of risky assets. Klein(2011) found that factors other than market and size factors play an important role in explaining portfolios' volatility. Adcock et al. (2019) found that a small number of variables, including the change in the term premium, unexpected inflation, and change in volatility dominate non-market explained variation.

2.2 Orthogonalization methods

We used two orthogonalization methods, namely, the Gram-Schmidt (GS) method and Schweinler–Wigner/Löwdin (SW/L) method, and compared their performance when the number of risk factors increases.

2.2.1 Gram-Schmidt process

The Gram–Schmidt process is a method for ortho-normalizing a set of vectors in an inner product space, most commonly the Euclidean space R_n equipped with the standard inner product. The process works as follows, starting with a set of vectors $\{v_i, i = 1, \dots, k\}$:

$$u_1 = v_1$$
$$u_2 = v_2 - proj_{u_1}(v_2)$$
$$u_3 = v_3 - proj_{u_1}(v_3) - proj_{u_2}(v_3)$$
$$\dots$$
$$u_k = v_k - \sum_{j=1}^{k-1} proj_{u_j}(v_k)$$

Here $proj_u(v)$ denotes the projection of the vector v to u.
The vectors $u_1, u_2, \dots, u_k$ are the orthogonal set of vectors generated in this process.

2.2.2 Schweinler–Wigner/Löwdin (1970) symmetric procedure

The Schweinler–Wigner procedure is a method for orthogonalizing a set of vectors (arranged as the columns of a matrix in this procedure) symmetrically. It has the advantage that it is not sensitive to the order in which vectors are arranged. The procedure maximizes the overlap between the original vectors and the orthogonalized vectors generated. The procedure treats all factors on an equal footing. The process works as follows:

F_{TK} denotes the original factor values from factor 1 to k and time 1 to T.
We want to find a matrix S_{KK} such that

$$G_{TK} = F_{TK} S_{KK},$$

where G_{TK} is the required set of orthogonal vectors. The orthonormal condition of G_{TK} means that $G^T G = I$, $S^T F^T F S = I$, or $S^T M S = I$, where I is the identity matrix and M is the Gram matrix of the original set.

By diagonalizing M, we can find O_{KK}, D_{KK}, such that

$$M_{KK} = O_{KK} D_{KK} O_{KK}^T,$$

It is easy to see that $S^T M S = I$ can be solved by constructing

$$S_{KK} = O_{KK} D_{KK}^{-1/2} O_{KK}^T$$

3. The US market

3.1 The 14-factor model

In this session, we consider a 14-factor risk model in the US stock market, which include the well-known Fama-French three factors, together with eleven sector factors based on Global Industry Classification Standard (GICS)– Energy, Materials, Industrials, Consumer Discretionary, Consumer Staples, Health Care, Financials, Information Technology, Communication Services, Utilities, and Real Estate. The universe we consider consists of the component stocks of the SP500 index. We use the monthly returns of stocks from January 2003 to May 2021 to construct the risk model. Note that the market component is removed from the sector returns, thus the correlation between the market risk factor and the sector risk factor is zero.

3.1.1 Orthogonalization performance

The tables below show the correlation between factors in the same set. We can see that the correlations are quite significant among the original factors; the orthogonalization eliminates the correlations among the orthogonalized factors as expected.

	Mkt-RF	SMB	HML	Energy	Materials	Industrials	Cons Disc	Cons Stap	Healthcare	Financials	IT	Comm	Utilities	Real Estate
Mkt-RF	1.00000	0.37367	0.26729	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
SMB	0.37367	1.00000	0.15744	0.10674	0.01603	-0.00670	0.15822	-0.43301	-0.14764	-0.09183	-0.03617	-0.21774	-0.21698	0.04008
HML	0.26729	0.15744	1.00000	0.22876	0.06446	0.28801	-0.14439	-0.05353	-0.21693	0.64774	-0.41968	-0.18264	-0.05173	0.16311
Energy	0.00000	0.10674	0.22876	1.00000	0.25446	-0.02856	-0.32608	-0.19353	-0.20117	-0.11960	-0.23034	-0.04288	0.00085	-0.17960
Materials	0.00000	0.01603	0.06446	0.25446	1.00000	0.28534	-0.03809	-0.17291	-0.18603	-0.04312	-0.03461	-0.07659	-0.11710	0.06241
Industrials	0.00000	-0.00670	0.28801	-0.02856	0.28534	1.00000	-0.01455	0.05419	-0.15669	0.24492	-0.20616	-0.11566	-0.09148	0.10085
Cons Disc	0.00000	0.15822	-0.14439	-0.32608	-0.03809	-0.01455	1.00000	-0.02854	-0.13787	-0.07257	0.18587	0.09596	-0.22896	0.19483
Cons Stap	0.00000	-0.43301	-0.05353	-0.19353	-0.17291	0.05419	-0.02854	1.00000	0.26097	-0.06079	-0.16230	0.14591	0.41183	0.13199
Healthcare	0.00000	-0.14764	-0.21693	-0.20117	-0.18603	-0.15669	-0.13787	0.26097	1.00000	-0.08644	-0.25697	-0.12670	0.12821	0.03628
Financials	0.00000	-0.09183	0.64774	-0.11960	-0.04312	0.24492	-0.07257	-0.06079	-0.08644	1.00000	-0.29951	-0.22419	-0.25551	0.10201
IT	0.00000	-0.03617	-0.41968	-0.23034	-0.03461	-0.20616	0.18587	-0.16230	-0.25697	-0.29951	1.00000	0.09079	-0.12590	-0.07676
Comm	0.00000	-0.21774	-0.18264	-0.04288	-0.07659	-0.11566	0.09596	0.14591	-0.12670	-0.22419	0.09079	1.00000	0.10573	-0.06461
Utilities	0.00000	-0.21698	-0.05173	0.00085	-0.11710	-0.09148	-0.22896	0.41183	0.12821	-0.25551	-0.12590	0.10573	1.00000	0.30151
Real Estate	0.00000	0.04008	0.16311	-0.17960	0.06241	0.10085	0.19483	0.13199	0.03628	0.10201	-0.07676	-0.06461	0.30151	1.00000

Table 1 Correlation of original factors

	Mkt-RF	SMB	HML	Energy	Materials	Industrials	Cons Disc	Cons Stap	Healthcare	Financials	IT	Comm	Utilities	Real Estate
Mkt-RF	1.00000	-0.00050	0.02084	-0.00487	-0.00016	-0.00737	0.00193	0.00032	0.00384	-0.02332	0.00840	0.00110	-0.00608	-0.00512
SMB	-0.00050	1.00000	0.00022	-0.00005	0.00000	-0.00008	0.00002	0.00000	0.00004	-0.00025	0.00009	0.00001	-0.00007	-0.00005
HML	0.02084	0.00022	1.00000	0.00218	0.00007	0.00330	-0.00086	-0.00014	-0.00172	0.01044	-0.00376	-0.00049	0.00272	0.00229
Energy	-0.00487	-0.00005	0.00218	1.00000	-0.00002	-0.00077	0.00020	0.00003	0.00040	-0.00244	0.00088	0.00011	-0.00063	-0.00053
Materials	-0.00016	0.00000	0.00007	-0.00002	1.00000	-0.00003	0.00001	0.00000	0.00001	-0.00008	0.00003	0.00000	-0.00002	-0.00002
Industrials	-0.00737	-0.00008	0.00330	-0.00077	-0.00003	1.00000	0.00030	0.00005	0.00061	-0.00369	0.00133	0.00017	-0.00096	-0.00081
Cons Disc	0.00193	0.00002	-0.00086	0.00020	0.00001	0.00030	1.00000	-0.00001	-0.00016	0.00096	-0.00035	-0.00005	0.00025	0.00021
Cons Stap	0.00032	0.00000	-0.00014	0.00003	0.00000	0.00005	-0.00001	1.00000	-0.00003	0.00016	-0.00006	-0.00001	0.00004	0.00003
Healthcare	0.00384	0.00004	-0.00172	0.00040	0.00001	0.00061	-0.00016	-0.00003	1.00000	0.00192	-0.00069	-0.00009	0.00050	0.00042
Financials	-0.02332	-0.00025	0.01044	-0.00244	-0.00008	-0.00369	0.00096	0.00016	0.00192	1.00000	0.00420	0.00055	-0.00304	-0.00256
IT	0.00840	0.00009	-0.00376	0.00088	0.00003	0.00133	-0.00035	-0.00006	-0.00069	0.00420	1.00000	-0.00020	0.00110	0.00092
Comm	0.00110	0.00001	-0.00049	0.00011	0.00000	0.00017	-0.00005	-0.00001	-0.00009	0.00055	-0.00020	1.00000	0.00014	0.00012
Utilities	-0.00608	-0.00007	0.00272	-0.00063	-0.00002	-0.00096	0.00025	0.00004	0.00050	-0.00304	0.00110	0.00014	1.00000	-0.00067
Real Estate	-0.00512	-0.00005	0.00229	-0.00053	-0.00002	-0.00081	0.00021	0.00003	0.00042	-0.00256	0.00092	0.00012	-0.00067	1.00000

Table 2 Correlation of orthogonalized factors using the GS method

	Mkt-RF	SMB	HML	Energy	Materials	Industrials	Cons Disc	Cons Stap	Healthcare	Financials	IT	Comm	Utilities	Real Estate
Mkt-RF	1.00000	-0.00782	0.01702	-0.00761	0.00017	-0.00574	0.00267	-0.00106	0.00705	-0.02311	0.00967	0.00211	-0.00104	-0.00481
SMB	-0.00782	1.00000	0.00270	-0.00121	0.00003	-0.00091	0.00042	-0.00017	0.00112	-0.00367	0.00153	0.00034	-0.00016	-0.00076
HML	0.01702	0.00270	1.00000	0.00263	-0.00006	0.00198	-0.00092	0.00037	-0.00243	0.00798	-0.00334	-0.00073	0.00036	0.00166
Energy	-0.00761	-0.00121	0.00263	1.00000	0.00003	-0.00089	0.00041	-0.00016	0.00109	-0.00357	0.00149	0.00033	-0.00016	-0.00074
Materials	0.00017	0.00003	-0.00006	0.00003	1.00000	0.00002	-0.00001	0.00000	-0.00002	0.00008	-0.00003	-0.00001	0.00000	0.00002
Industrials	-0.00574	-0.00091	0.00198	-0.00089	0.00002	1.00000	0.00031	-0.00012	0.00082	-0.00269	0.00113	0.00025	-0.00012	-0.00056
Cons Disc	0.00267	0.00042	-0.00092	0.00041	-0.00001	0.00031	1.00000	0.00006	-0.00038	0.00125	-0.00052	-0.00011	0.00006	0.00026
Cons Stap	-0.00106	-0.00017	0.00037	-0.00016	0.00000	-0.00012	0.00006	1.00000	0.00015	-0.00050	0.00021	0.00005	-0.00002	-0.00010
Healthcare	0.00705	0.00112	-0.00243	0.00109	-0.00002	0.00082	-0.00038	0.00015	1.00000	0.00330	-0.00138	-0.00030	0.00015	0.00069
Financials	-0.02311	-0.00367	0.00798	-0.00357	0.00008	-0.00269	0.00125	-0.00050	0.00330	1.00000	0.00453	0.00099	-0.00049	-0.00225
IT	0.00967	0.00153	-0.00334	0.00149	-0.00003	0.00113	-0.00052	0.00021	-0.00138	0.00453	1.00000	-0.00041	0.00020	0.00094
Comm	0.00211	0.00034	-0.00073	0.00033	-0.00001	0.00025	-0.00011	0.00005	-0.00030	0.00099	-0.00041	1.00000	0.00004	0.00021
Utilities	-0.00104	-0.00016	0.00036	-0.00016	0.00000	-0.00012	0.00006	-0.00002	0.00015	-0.00049	0.00020	0.00004	1.00000	-0.00010
Real Estate	-0.00481	-0.00076	0.00166	-0.00074	0.00002	-0.00056	0.00026	-0.00010	0.00069	-0.00225	0.00094	0.00021	-0.00010	1.00000

Table 3 Correlation of orthogonalized factors using the SW/L method

What is important for our purpose is whether the orthogonalized factors are closely related to the original as we aim to retain the most economic intuition of the original factors. The table below shows the correlation between original and orthogonalized factors using the GS and SW/L schemes.

	Mkt-RF	SMB	HML	Energy	Materials	Industrials	Cons Disc	Cons Stap	Healthcare	Financials	IT	Comm	Utilities	Real Estate
GS	1.00000	0.92738	0.96699	0.96676	0.96698	0.90096	0.91676	0.85755	0.90186	0.66067	0.76594	0.89659	0.81304	0.84494
SW/L	0.98524	0.96791	0.98749	0.94171	0.96822	0.92781	0.92397	0.83846	0.91628	0.68904	0.86180	0.94235	0.92578	0.95786

Table 4 Correlation between original and orthogonalized factors

We can see that the SW/L method outperforms the GS method in this measure. Using the GS method, six correlation coefficients are below 0.9 whereas using the SW/L method, three correlation coefficients are below 0.9, with the lowest value being 0.69 for the Financials factor. The low value of the correlation for the financials factor is due to the high correlation of the financials factor to other factors in the original set (for example, the financials factor has a high correlation value of 0.65 with HML). But overall the SW/L method is a better orthogonalization scheme for generating a bigger overlap of the original and the orthogonalized factors.

3.1.2 Comparing the SW/L method over multiple periods

For the SW/L method, we would like to study the stability of the coefficient matrix relating old factors to new factors over multiple periods. This is important for a robust risk model construction using the orthogonalization scheme. Over the two 5-year periods (2011-2015, 2016-2020), we check that the coefficient matrix is indeed quite stable. The correlation coefficient between $S_{2011-2015}$ and $S_{2016-2020}$ is 0.9249. The orthogonalized factors generated from the SW/L method are thus stable over the two five-year periods.

Regressing the performance of individual stocks on the fourteen factors, the R^2 is 0.4856 on average. The systematic risk accounts for 48.56% of the total variance in stock performance.

3.2 The 27-factor model

To construct the 27-factor model, we chose the Fama-French three factors, together with twenty-four industry group factors based on Global Industry Classification Standard (GICS). we used the GICS code to represent the industry group as shown in Table 5. Note that the market component is removed from the industry group returns

Industry Group	
1010	Energy
1510	Materials
2010	Capital Goods
2020	Commercial & Professional Services
2030	Transportation
2510	Automobiles & Components
2520	Consumer Durables & Apparel
2530	Consumer Services
2550	Retailing
3010	Food & Staples Retailing
3020	Food, Beverage & Tobacco
3030	Household & Personal Products
3510	Health Care Equipment & Services
3520	Pharmaceuticals, Biotechnology & Life Sciences
4010	Banks
4020	Diversified Financials
4030	Insurance
4510	Software & Services
4520	Technology Hardware & Equipment
4530	Semiconductors & Semiconductor Equipment
5010	Communication Services
5020	Media & Entertainment
5510	Utilities
6010	Real Estate

Table 5 GICS industry group code

3.2.1 Orthogonalization performance

The tables below show the correlation between factors in the same set. We can see that the correlation is mostly eliminated after orthogonalization using both methods.

	Mkt-RF	SMB	HML	1010	1510	2010	2020	2030	2510	2520	2530	2550	3010	3020	3030	3510	3520	4010	4020	4030	4510	4520	4530	5010	5020	5510	6010
Mkt-RF	1.00000	0.37367	0.26729	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
SMB	0.37367	1.00000	0.15744	0.10674	0.01603	-0.01874	-0.02482	0.07177	0.08991	0.18553	0.03801	0.10996	-0.23366	-0.37915	-0.31862	-0.01546	-0.18208	-0.04461	-0.09519	-0.11972	-0.18334	0.00218	0.15221	-0.22021	-0.14248	-0.21698	0.04008
HML	0.26729	0.15744	1.00000	0.22876	0.06446	0.29999	-0.09345	0.16742	0.01031	0.19045	0.07457	-0.21337	-0.07700	-0.01428	-0.03961	-0.17212	-0.19443	0.66561	0.40644	0.41337	-0.38961	-0.31174	-0.18672	-0.00830	-0.19059	-0.05173	0.16311
1010	0.00000	0.10674	0.22876	1.00000	0.25446	0.05212	-0.07303	-0.17601	-0.13968	-0.17161	-0.03090	-0.32295	-0.27770	-0.04935	-0.16309	-0.14900	-0.19073	-0.10552	-0.08212	-0.09399	-0.11961	-0.24431	-0.13093	0.05716	-0.08537	0.00085	-0.17960
1510	0.00000	0.01603	0.06446	0.25446	1.00000	0.28178	-0.03755	0.17462	0.15705	0.03393	0.10902	-0.12670	-0.07250	-0.16552	-0.16463	-0.15217	-0.17371	-0.10964	0.08524	0.03438	-0.03524	-0.09115	0.08319	0.00391	-0.06044	-0.11710	0.06241
2010	0.00000	-0.01874	0.29999	0.05212	0.28178	1.00000	0.09483	0.31663	0.10092	0.15821	0.22980	-0.20671	-0.02279	-0.02817	0.08613	-0.11291	-0.12282	0.22687	0.12143	0.28075	-0.15626	-0.22037	-0.05074	0.04988	-0.16448	-0.10500	0.08112
2020	0.00000	-0.02482	-0.09345	-0.07303	-0.03755	0.09483	1.00000	0.14441	-0.04269	-0.01455	0.04714	-0.05936	0.16545	0.18656	0.19373	0.04000	-0.06985	-0.16228	-0.06326	0.10064	0.00205	-0.08148	-0.06107	0.18485	0.01445	0.27818	0.11373
2030	0.00000	0.07177	0.16742	-0.17601	0.17462	0.31663	0.14441	1.00000	0.10174	0.24191	0.21009	0.05402	0.23916	-0.04611	0.06340	-0.19049	-0.08281	0.18430	0.15013	0.03214	-0.10475	-0.14371	0.05379	0.06400	-0.09176	-0.10716	0.06731
2510	0.00000	0.08991	0.01031	-0.13968	0.15705	0.10092	-0.04269	0.10174	1.00000	0.24334	0.19435	0.14794	0.01710	-0.16054	-0.04042	-0.23490	-0.23749	0.11894	0.13060	0.02568	0.07460	0.17114	-0.01333	-0.05973	0.03895	-0.15084	0.13328
2520	0.00000	0.18553	0.19045	-0.17161	0.03393	0.15821	-0.01455	0.24191	0.24334	1.00000	0.30083	0.36030	0.11459	-0.04914	0.08462	-0.10539	-0.21059	0.17336	0.07676	0.17657	-0.17623	-0.04197	0.00829	-0.09358	-0.13023	-0.07449	0.31103
2530	0.00000	0.03801	0.07457	-0.03090	0.10902	0.22980	0.04714	0.21009	0.19435	0.30083	1.00000	0.06226	-0.02095	0.03405	0.07828	-0.08591	-0.16718	0.11437	0.07789	0.15770	0.04160	-0.06286	-0.05716	0.00075	0.20234	-0.09647	0.23521
2550	0.00000	0.10996	-0.21337	-0.32295	-0.12670	-0.20671	-0.05936	0.05402	0.14794	0.36030	0.06226	1.00000	0.26536	-0.17020	-0.10363	0.02075	-0.02663	-0.14411	-0.16748	-0.12994	0.13443	0.14294	0.19224	-0.09179	0.16228	-0.18465	0.09089
3010	0.00000	-0.23366	-0.07700	-0.27770	-0.07250	-0.02279	0.16545	0.23916	0.01710	0.11459	-0.02095	0.26536	1.00000	0.17615	0.22859	0.03982	0.14154	-0.00957	0.03550	0.01649	-0.03448	-0.06546	-0.02923	0.21772	-0.04648	0.13355	0.04869
3020	0.00000	-0.37915	-0.01428	-0.04935	-0.16552	-0.02817	0.18656	-0.04611	-0.16054	-0.04914	0.03405	-0.17020	0.17615	1.00000	0.56148	0.15609	0.23221	-0.08721	-0.03985	-0.00137	-0.10154	-0.08535	-0.20324	0.32189	0.08494	0.43118	0.10030
3030	0.00000	-0.31862	-0.03961	-0.16309	-0.16463	0.08613	0.19373	0.06340	-0.04042	0.08462	0.07828	-0.10363	0.22859	0.56148	1.00000	0.11277	0.27623	-0.05046	-0.06616	0.01886	-0.11010	-0.06158	-0.16749	0.21259	-0.07554	0.30875	0.15771
3510	0.00000	-0.01546	-0.17212	-0.14900	-0.15217	-0.11291	0.04000	-0.19049	-0.23490	-0.10539	-0.08591	0.02075	0.03982	0.15609	0.11277	1.00000	0.45449	-0.08867	-0.10604	-0.08452	-0.08965	-0.15266	-0.19043	-0.09758	-0.07507	0.10351	0.06876
3520	0.00000	-0.18208	-0.19443	-0.19073	-0.17371	-0.12282	-0.06985	-0.08281	-0.23749	-0.21059	-0.16718	-0.02663	0.14154	0.23221	0.27623	0.45449	1.00000	-0.03638	-0.10304	-0.07647	-0.11290	-0.22038	-0.18268	0.11827	-0.17635	0.11339	0.00846
4010	0.00000	-0.04461	0.66561	-0.10552	-0.10964	0.22687	-0.16228	0.18430	0.11894	0.17336	0.11437	-0.14411	-0.00957	-0.08721	-0.05046	-0.08867	-0.03638	1.00000	0.58675	0.51472	-0.22412	-0.25689	-0.18654	-0.17699	-0.15850	-0.29139	0.11030
4020	0.00000	-0.09519	0.40644	-0.08212	0.08524	0.12143	-0.06326	0.15013	0.13060	0.07676	0.07789	-0.16748	0.03550	-0.03985	-0.06616	-0.10604	-0.10304	0.58675	1.00000	0.33249	0.00319	-0.13228	-0.10484	-0.11465	-0.12109	-0.19720	-0.02445
4030	0.00000	-0.11972	0.41337	-0.09399	0.03438	0.28075	0.10064	0.03214	0.02568	0.17657	0.15770	-0.12994	0.01649	-0.00137	0.01886	-0.08452	-0.07647	0.51472	0.33249	1.00000	-0.15486	-0.26336	-0.18332	-0.02253	-0.14467	0.00959	0.19299
4510	0.00000	-0.18334	-0.38961	-0.11961	-0.03524	-0.15626	0.00205	-0.10475	0.07460	-0.17623	0.04160	0.13443	-0.03448	-0.10154	-0.11010	-0.08965	-0.11290	-0.22412	0.00319	-0.15486	1.00000	0.33535	0.17150	0.03445	0.23961	-0.06340	-0.10115
4520	0.00000	0.00218	-0.31174	-0.24431	-0.09115	-0.22037	-0.08148	-0.14371	0.17114	-0.04197	-0.06286	0.14294	-0.06546	-0.08535	-0.06158	-0.15266	-0.22038	-0.25689	-0.13228	-0.26336	0.33535	1.00000	0.41650	-0.08911	0.05306	-0.05503	-0.01829
4530	0.00000	0.15221	-0.18672	-0.13093	0.08319	-0.05074	-0.06107	0.05379	-0.01333	0.00829	-0.05716	0.19224	-0.02923	-0.20324	-0.16749	-0.19043	-0.18268	-0.18654	-0.10484	-0.18332	0.17150	0.41650	1.00000	-0.19977	0.02428	-0.16995	-0.07809
5010	0.00000	-0.22021	-0.00830	0.05716	0.00391	0.04988	0.18485	0.06400	-0.05973	-0.09358	0.00075	-0.09179	0.21772	0.32189	0.21259	-0.09758	0.11827	-0.17699	-0.11465	-0.02253	0.03445	-0.08911	-0.19977	1.00000	0.00527	0.33872	0.05139
5020	0.00000	-0.14248	-0.19059	-0.08537	-0.06044	-0.16448	0.01445	-0.09176	0.03895	-0.13023	0.20234	0.16228	-0.04648	0.08494	-0.07554	-0.07507	-0.17635	-0.15850	-0.12109	-0.14467	0.23961	0.05306	0.02428	0.00527	1.00000	-0.03081	-0.04089
5510	0.00000	-0.21698	-0.05173	0.00085	-0.11710	-0.10500	0.27818	-0.10716	-0.15084	-0.07449	-0.09647	-0.18465	0.13355	0.43118	0.30875	0.10351	0.11339	-0.29139	-0.19720	0.00959	-0.06340	-0.05503	-0.16995	0.33872	-0.03081	1.00000	0.30151
6010	0.00000	0.04008	0.16311	-0.17960	0.06241	0.08112	0.11373	0.06731	0.13328	0.31103	0.23521	0.09089	0.04869	0.10030	0.15771	0.06876	0.00846	0.11030	-0.02445	0.19299	-0.10115	-0.01829	-0.07809	0.05139	-0.04089	0.30151	1.00000

Table 6 Correlation of original factors

	Mkt-RF	SMB	HML	1010	1510	2010	2020	2030	2510	2520	2530	2550	3010	3020	3030	3510	3520	4010	4020	4030	4510	4520	4530	5010	5020	5510	6010
Mkt-RF	1.00000	-0.00050	0.02084	-0.00487	-0.00016	-0.00718	0.00277	-0.00372	-0.00027	-0.00419	0.00102	0.00509	-0.00017	0.00001	0.00208	0.00218	0.00215	-0.02446	-0.00125	-0.00404	0.00990	0.00114	0.00173	-0.00620	0.00046	-0.00853	-0.00439
SMB	-0.00050	1.00000	0.00022	-0.00005	0.00000	-0.00008	0.00003	-0.00004	0.00000	-0.00004	0.00001	0.00005	0.00000	0.00000	0.00002	0.00002	0.00002	-0.00026	-0.00001	-0.00004	0.00011	0.00001	0.00002	-0.00007	0.00000	-0.00009	-0.00005
HML	0.02084	0.00022	1.00000	0.00218	0.00007	0.00321	-0.00124	0.00166	0.00012	0.00188	-0.00046	-0.00228	0.00008	-0.00001	-0.00093	-0.00098	-0.00096	0.01094	0.00056	0.00181	-0.00443	-0.00051	-0.00077	0.00277	-0.00021	0.00382	0.00197
1010	-0.00487	-0.00005	0.00218	1.00000	-0.00002	-0.00075	0.00029	-0.00039	-0.00003	-0.00044	0.00011	0.00053	-0.00002	0.00000	0.00022	0.00023	0.00023	-0.00255	-0.00013	-0.00042	0.00103	0.00012	0.00018	-0.00065	0.00005	-0.00089	-0.00046
1510	-0.00016	0.00000	0.00007	-0.00002	1.00000	-0.00003	0.00001	-0.00001	0.00000	-0.00001	0.00000	0.00002	0.00000	0.00000	0.00001	0.00001	0.00001	-0.00009	0.00000	-0.00001	0.00003	0.00000	0.00001	-0.00002	0.00000	-0.00003	-0.00002
2010	-0.00718	-0.00008	0.00321	-0.00075	-0.00003	1.00000	0.00043	-0.00057	-0.00004	-0.00065	0.00016	0.00078	-0.00003	0.00000	0.00032	0.00034	0.00033	-0.00377	-0.00019	-0.00062	0.00153	0.00018	0.00027	-0.00095	0.00007	-0.00131	-0.00068
2020	0.00277	0.00003	-0.00124	0.00029	0.00001	0.00043	1.00000	0.00022	0.00002	0.00025	-0.00006	-0.00030	0.00001	0.00000	-0.00012	-0.00013	-0.00013	0.00145	0.00007	0.00024	-0.00059	-0.00007	-0.00010	0.00037	-0.00003	0.00051	0.00026
2030	-0.00372	-0.00004	0.00166	-0.00039	-0.00001	-0.00057	0.00022	1.00000	-0.00002	-0.00033	0.00008	0.00041	-0.00001	0.00000	0.00017	0.00017	0.00017	-0.00195	-0.00010	-0.00032	0.00079	0.00009	0.00014	-0.00049	0.00004	-0.00068	-0.00035
2510	-0.00027	0.00000	0.00012	-0.00003	0.00000	-0.00004	0.00002	-0.00002	1.00000	-0.00002	0.00001	0.00003	0.00000	0.00000	0.00001	0.00001	0.00001	-0.00014	-0.00001	-0.00002	0.00006	0.00001	0.00001	-0.00004	0.00000	-0.00005	-0.00003
2520	-0.00419	-0.00004	0.00188	-0.00044	-0.00001	-0.00065	0.00025	-0.00033	-0.00002	1.00000	0.00009	0.00046	-0.00002	0.00000	0.00019	0.00020	0.00019	-0.00220	-0.00011	-0.00036	0.00089	0.00010	0.00016	-0.00056	0.00004	-0.00077	-0.00040
2530	0.00102	0.00001	-0.00046	0.00011	0.00000	0.00016	-0.00006	0.00008	0.00001	0.00009	1.00000	-0.00011	0.00000	0.00000	-0.00005	-0.00005	-0.00005	0.00053	0.00003	0.00009	-0.00022	-0.00003	-0.00004	0.00014	-0.00001	0.00019	0.00010
2550	0.00509	0.00005	-0.00228	0.00053	0.00002	0.00078	-0.00030	0.00041	0.00003	0.00046	-0.00011	1.00000	0.00002	0.00000	-0.00023	-0.00024	-0.00024	0.00267	0.00014	0.00044	-0.00108	-0.00012	-0.00019	0.00068	-0.00005	0.00093	0.00048
3010	-0.00017	0.00000	0.00008	-0.00002	0.00000	-0.00003	0.00001	-0.00001	0.00000	-0.00002	0.00000	0.00002	1.00000	0.00000	0.00001	0.00001	0.00001	-0.00009	0.00000	-0.00001	0.00004	0.00000	0.00001	-0.00002	0.00000	-0.00003	-0.00002
3020	0.00001	0.00000	-0.00001	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000	0.00000	0.00000	0.00000	0.00001	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
3030	0.00208	0.00002	-0.00093	0.00022	0.00001	0.00032	-0.00012	0.00017	0.00001	0.00019	-0.00005	-0.00023	0.00001	0.00000	1.00000	-0.00010	-0.00010	0.00109	0.00006	0.00018	-0.00044	-0.00005	-0.00008	0.00028	-0.00002	0.00038	0.00020
3510	0.00218	0.00002	-0.00098	0.00023	0.00001	0.00034	-0.00013	0.00017	0.00001	0.00020	-0.00005	-0.00024	0.00001	0.00000	-0.00010	1.00000	-0.00010	0.00115	0.00006	0.00019	-0.00046	-0.00005	-0.00008	0.00029	-0.00002	0.00040	0.00021
3520	0.00215	0.00002	-0.00096	0.00023	0.00001	0.00033	-0.00013	0.00017	0.00001	0.00019	-0.00005	-0.00024	0.00001	0.00000	-0.00010	-0.00010	1.00000	0.00113	0.00006	0.00019	-0.00046	-0.00005	-0.00008	0.00029	-0.00002	0.00039	0.00020
4010	-0.02446	-0.00026	0.01094	-0.00255	-0.00009	-0.00377	0.00145	-0.00195	-0.00014	-0.00220	0.00053	0.00267	-0.00009	0.00001	0.00109	0.00115	0.00113	1.00000	-0.00066	-0.00212	0.00520	0.00060	0.00091	-0.00325	0.00024	-0.00448	-0.00231
4020	-0.00125	-0.00001	0.00056	-0.00013	0.00000	-0.00019	0.00007	-0.00010	-0.00001	-0.00011	0.00003	0.00014	0.00000	0.00000	0.00006	0.00006	0.00006	-0.00066	1.00000	-0.00011	0.00027	0.00003	0.00005	-0.00017	0.00001	-0.00023	-0.00012
4030	-0.00404	-0.00004	0.00181	-0.00042	-0.00001	-0.00062	0.00024	-0.00032	-0.00002	-0.00036	0.00009	0.00044	-0.00001	0.00000	0.00018	0.00019	0.00019	-0.00212	-0.00011	1.00000	0.00086	0.00010	0.00015	-0.00054	0.00004	-0.00074	-0.00038
4510	0.00990	0.00011	-0.00443	0.00103	0.00003	0.00153	-0.00059	0.00079	0.00006	0.00089	-0.00022	-0.00108	0.00004	0.00000	-0.00044	-0.00046	-0.00046	0.00520	0.00027	0.00086	1.00000	-0.00024	-0.00037	0.00132	-0.00010	0.00181	0.00093
4520	0.00114	0.00001	-0.00051	0.00012	0.00000	0.00018	-0.00007	0.00009	0.00001	0.00010	-0.00003	-0.00012	0.00000	0.00000	-0.00005	-0.00005	-0.00005	0.00060	0.00003	0.00010	-0.00024	1.00000	-0.00004	0.00015	-0.00001	0.00021	0.00011
4530	0.00173	0.00002	-0.00077	0.00018	0.00001	0.00027	-0.00010	0.00014	0.00001	0.00016	-0.00004	-0.00019	0.00001	0.00000	-0.00008	-0.00008	-0.00008	0.00091	0.00005	0.00015	-0.00037	-0.00004	1.00000	0.00023	-0.00002	0.00032	0.00016
5010	-0.00620	-0.00007	0.00277	-0.00065	-0.00002	-0.00095	0.00037	-0.00049	-0.00004	-0.00056	0.00014	0.00068	-0.00002	0.00000	0.00028	0.00029	0.00029	-0.00325	-0.00017	-0.00054	0.00132	0.00015	0.00023	1.00000	0.00006	-0.00114	-0.00058
5020	0.00046	0.00000	-0.00021	0.00005	0.00000	0.00007	-0.00003	0.00004	0.00000	0.00004	-0.00001	-0.00005	0.00000	0.00000	-0.00002	-0.00002	-0.00002	0.00024	0.00001	0.00004	-0.00010	-0.00001	-0.00002	0.00006	1.00000	0.00008	0.00004
5510	-0.00853	-0.00009	0.00382	-0.00089	-0.00003	-0.00131	0.00051	-0.00068	-0.00005	-0.00077	0.00019	0.00093	-0.00003	0.00000	0.00038	0.00040	0.00039	-0.00448	-0.00023	-0.00074	0.00181	0.00021	0.00032	-0.00114	0.00008	1.00000	-0.00080
6010	-0.00439	-0.00005	0.00197	-0.00046	-0.00002	-0.00068	0.00026	-0.00035	-0.00003	-0.00040	0.00010	0.00048	-0.00002	0.00000	0.00020	0.00021	0.00020	-0.00231	-0.00012	-0.00038	0.00093	0.00011	0.00016	-0.00058	0.00004	-0.00080	1.00000

Table 7 Correlation of orthogonalized factors using the GS method

	Mkt-RF	SMB	HML	1010	1510	2010	2020	2030	2510	2520	2530	2550	3010	3020	3030	3510	3520	4010	4020	4030	4510	4520	4530	5010	5020	5510	6010
Mkt-RF	1.00000	-0.00785	0.01729	-0.00924	-0.00141	-0.00580	0.00305	-0.00303	0.00127	-0.00320	0.00140	0.00190	0.00142	-0.00157	-0.00020	0.00347	0.00694	-0.02382	-0.00645	-0.00635	0.01008	0.00413	0.00222	-0.00272	0.00176	-0.00329	-0.00444
SMB	-0.00785	1.00000	0.00275	-0.00147	-0.00022	-0.00092	0.00049	-0.00048	0.00020	-0.00051	0.00022	0.00030	0.00023	-0.00025	-0.00003	0.00055	0.00110	-0.00379	-0.00103	-0.00101	0.00160	0.00066	0.00035	-0.00043	0.00028	-0.00052	-0.00071
HML	0.01729	0.00275	1.00000	0.00324	0.00049	0.00203	-0.00107	0.00106	-0.00044	0.00112	-0.00049	-0.00066	-0.00050	0.00055	0.00007	-0.00122	-0.00243	0.00834	0.00226	0.00222	-0.00353	-0.00145	-0.00078	0.00095	-0.00062	0.00115	0.00155
1010	-0.00924	-0.00147	0.00324	1.00000	-0.00026	-0.00109	0.00057	-0.00057	0.00024	-0.00060	0.00026	0.00036	0.00027	-0.00029	-0.00004	0.00065	0.00130	-0.00446	-0.00121	-0.00119	0.00189	0.00077	0.00042	-0.00051	0.00033	-0.00062	-0.00083
1510	-0.00141	-0.00022	0.00049	-0.00026	1.00000	-0.00017	0.00009	-0.00009	0.00004	-0.00009	0.00004	0.00005	0.00004	-0.00004	-0.00001	0.00010	0.00020	-0.00068	-0.00018	-0.00018	0.00029	0.00012	0.00006	-0.00008	0.00005	-0.00009	-0.00013
2010	-0.00580	-0.00092	0.00203	-0.00109	-0.00017	1.00000	0.00036	-0.00036	0.00015	-0.00038	0.00016	0.00022	0.00017	-0.00018	-0.00002	0.00041	0.00081	-0.00280	-0.00076	-0.00075	0.00118	0.00048	0.00026	-0.00032	0.00021	-0.00039	-0.00052
2020	0.00305	0.00049	-0.00107	0.00057	0.00009	0.00036	1.00000	0.00019	-0.00008	0.00020	-0.00009	-0.00012	-0.00009	0.00010	0.00001	-0.00021	-0.00043	0.00147	0.00040	0.00039	-0.00062	-0.00026	-0.00014	0.00017	-0.00011	0.00020	0.00027
2030	-0.00303	-0.00048	0.00106	-0.00057	-0.00009	-0.00036	0.00019	1.00000	0.00008	-0.00020	0.00009	0.00012	0.00009	-0.00010	-0.00001	0.00021	0.00043	-0.00146	-0.00040	-0.00039	0.00062	0.00025	0.00014	-0.00017	0.00011	-0.00020	-0.00027
2510	0.00127	0.00020	-0.00044	0.00024	0.00004	0.00015	-0.00008	0.00008	1.00000	0.00008	-0.00004	-0.00005	-0.00004	0.00004	0.00001	-0.00009	-0.00018	0.00061	0.00017	0.00016	-0.00026	-0.00011	-0.00006	0.00007	-0.00005	0.00008	0.00011
2520	-0.00320	-0.00051	0.00112	-0.00060	-0.00009	-0.00038	0.00020	-0.00020	0.00008	1.00000	0.00009	0.00012	0.00009	-0.00010	-0.00001	0.00023	0.00045	-0.00155	-0.00042	-0.00041	0.00065	0.00027	0.00014	-0.00018	0.00011	-0.00021	-0.00029
2530	0.00140	0.00022	-0.00049	0.00026	0.00004	0.00016	-0.00009	0.00009	-0.00004	0.00009	1.00000	-0.00005	-0.00004	0.00004	0.00001	-0.00010	-0.00020	0.00067	0.00018	0.00018	-0.00029	-0.00012	-0.00006	0.00008	-0.00005	0.00009	0.00013
2550	0.00190	0.00030	-0.00066	0.00036	0.00005	0.00022	-0.00012	0.00012	-0.00005	0.00012	-0.00005	1.00000	-0.00005	0.00006	0.00001	-0.00013	-0.00027	0.00092	0.00025	0.00024	-0.00039	-0.00016	-0.00009	0.00010	-0.00007	0.00013	0.00017
3010	0.00142	0.00023	-0.00050	0.00027	0.00004	0.00017	-0.00009	0.00009	-0.00004	0.00009	-0.00004	-0.00005	1.00000	0.00005	0.00001	-0.00010	-0.00020	0.00068	0.00019	0.00018	-0.00029	-0.00012	-0.00006	0.00008	-0.00005	0.00009	0.00013
3020	-0.00157	-0.00025	0.00055	-0.00029	-0.00004	-0.00018	0.00010	-0.00010	0.00004	-0.00010	0.00004	0.00006	0.00005	1.00000	-0.00001	0.00011	0.00022	-0.00076	-0.00021	-0.00020	0.00032	0.00013	0.00007	-0.00009	0.00006	-0.00010	-0.00014
3030	-0.00020	-0.00003	0.00007	-0.00004	-0.00001	-0.00002	0.00001	-0.00001	0.00001	-0.00001	0.00001	0.00001	0.00001	-0.00001	1.00000	0.00001	0.00003	-0.00009	-0.00003	-0.00003	0.00004	0.00002	0.00001	-0.00001	0.00001	-0.00001	-0.00002
3510	0.00347	0.00055	-0.00122	0.00065	0.00010	0.00041	-0.00021	0.00021	-0.00009	0.00023	-0.00010	-0.00013	-0.00010	0.00011	0.00001	1.00000	-0.00049	0.00167	0.00045	0.00045	-0.00071	-0.00029	-0.00016	0.00019	-0.00012	0.00023	0.00031
3520	0.00694	0.00110	-0.00243	0.00130	0.00020	0.00081	-0.00043	0.00043	-0.00018	0.00045	-0.00020	-0.00027	-0.00020	0.00022	0.00003	-0.00049	1.00000	0.00335	0.00091	0.00089	-0.00142	-0.00058	-0.00031	0.00038	-0.00025	0.00046	0.00062
4010	-0.02382	-0.00379	0.00834	-0.00446	-0.00068	-0.00280	0.00147	-0.00146	0.00061	-0.00155	0.00067	0.00092	0.00068	-0.00076	-0.00009	0.00167	0.00335	1.00000	-0.00311	-0.00306	0.00486	0.00199	0.00107	-0.00131	0.00085	-0.00159	-0.00214
4020	-0.00645	-0.00103	0.00226	-0.00121	-0.00018	-0.00076	0.00040	-0.00040	0.00017	-0.00042	0.00018	0.00025	0.00019	-0.00021	-0.00003	0.00045	0.00091	-0.00311	1.00000	-0.00083	0.00132	0.00054	0.00029	-0.00036	0.00023	-0.00043	-0.00058
4030	-0.00635	-0.00101	0.00222	-0.00119	-0.00018	-0.00075	0.00039	-0.00039	0.00016	-0.00041	0.00018	0.00024	0.00018	-0.00020	-0.00003	0.00045	0.00089	-0.00306	-0.00083	1.00000	0.00130	0.00053	0.00029	-0.00035	0.00023	-0.00042	-0.00057
4510	0.01008	0.00160	-0.00353	0.00189	0.00029	0.00118	-0.00062	0.00062	-0.00026	0.00065	-0.00029	-0.00039	-0.00029	0.00032	0.00004	-0.00071	-0.00142	0.00486	0.00132	0.00130	1.00000	-0.00084	-0.00045	0.00055	-0.00036	0.00067	0.00091
4520	0.00413	0.00066	-0.00145	0.00077	0.00012	0.00048	-0.00026	0.00025	-0.00011	0.00027	-0.00012	-0.00016	-0.00012	0.00013	0.00002	-0.00029	-0.00058	0.00199	0.00054	0.00053	-0.00084	1.00000	-0.00019	0.00023	-0.00015	0.00027	0.00037
4530	0.00222	0.00035	-0.00078	0.00042	0.00006	0.00026	-0.00014	0.00014	-0.00006	0.00014	-0.00006	-0.00009	-0.00006	0.00007	0.00001	-0.00016	-0.00031	0.00107	0.00029	0.00029	-0.00045	-0.00019	1.00000	0.00012	-0.00008	0.00015	0.00020
5010	-0.00272	-0.00043	0.00095	-0.00051	-0.00008	-0.00032	0.00017	-0.00017	0.00007	-0.00018	0.00008	0.00010	0.00008	-0.00009	-0.00001	0.00019	0.00038	-0.00131	-0.00036	-0.00035	0.00055	0.00023	0.00012	1.00000	0.00010	-0.00018	-0.00024
5020	0.00176	0.00028	-0.00062	0.00033	0.00005	0.00021	-0.00011	0.00011	-0.00005	0.00011	-0.00005	-0.00007	-0.00005	0.00006	0.00001	-0.00012	-0.00025	0.00085	0.00023	0.00023	-0.00036	-0.00015	-0.00008	0.00010	1.00000	0.00012	0.00016
5510	-0.00329	-0.00052	0.00115	-0.00062	-0.00009	-0.00039	0.00020	-0.00020	0.00008	-0.00021	0.00009	0.00013	0.00009	-0.00010	-0.00001	0.00023	0.00046	-0.00159	-0.00043	-0.00042	0.00067	0.00027	0.00015	-0.00018	0.00012	1.00000	-0.00030
6010	-0.00444	-0.00071	0.00155	-0.00083	-0.00013	-0.00052	0.00027	-0.00027	0.00011	-0.00029	0.00013	0.00017	0.00013	-0.00014	-0.00002	0.00031	0.00062	-0.00214	-0.00058	-0.00057	0.00091	0.00037	0.00020	-0.00024	0.00016	-0.00030	1.00000

Table 8 Correlation of orthogonalized factors using the SW/L method

The overlap between the orthogonalized factors and the original factors is shown n the table below.

	Mkt-RF	SMB	HML	1010	1510	2010	2020	2030	2510	2520	2530	2550	3010	3020
GS	1.00000	0.92738	0.96699	0.96676	0.96698	0.90958	0.98369	0.89279	0.95957	0.89999	0.92202	0.83493	0.86390	0.85365
SW/L	0.98524	0.96786	0.98743	0.91235	0.93689	0.89598	0.94533	0.93319	0.98328	0.89028	0.94917	0.90159	0.92406	0.83619
	3030	3510	3520	4010	4020	4030	4510	4520	4530	5010	5020	5510	6010	
GS	0.77605	0.91538	0.79780	0.61474	0.77267	0.77686	0.81232	0.79032	0.83513	0.84665	0.82152	0.75518	0.80469	
SW/L	0.87801	0.90657	0.87792	0.67107	0.83082	0.85872	0.85947	0.89215	0.92875	0.93819	0.93025	0.88847	0.94178	

Table 9 Correlation between original and orthogonalized factors

Again the SW/L method outperforms the GS method in this overlap measure. Using the GS method, out of the twenty-seven correlations between new and original industry group factors, seven correlations are below 0.80 with the lowest value being 0.61.

Using the SW/L method, twenty-six out of twenty-seven-factor correlations are above 0.80, with the only exception being 0.67 for the Banks industry group (4010). These are likely dues to the relatively high correlation between Banks and other factors. Overall the outperformance of the SW/L is more pronounced using this larger set of factors.

3.2.2 Comparing the SW/L method over multiple periods

Using the SW/L method, we further investigate whether the coefficient matrix relating old factors to new factors is similar over multiple periods.
Over the two 5-year periods (2011-2015, 2016-2020), the coefficient matrix is stable. The correlation coefficient between $S_{2011\text{-}2015}$ and $S_{2016\text{-}2020}$ is 0.9498. On average, 57.29% of the total variance can be explained by the model, which is an improvement compared to the 14-factor model.

4. The China market

4.1 The 14-factor model

We perform a similar study in the context of the Chinese market, using the monthly returns of stocks from January 2006 to May 2021 May. The universe used consists of the component stocks for CSI 300. As for the US market, the fourteen factors are the Fama-French three factors plus eleven sector factors based on Global Industry Classification Standard (GICS).

4.1.1 Orthogonalization performance

The tables below show the correlation between factors in the same set.

	Mkt-RF	SMB	HML	Energy	Materials	Industrials	Cons Disc	Cons Stap	Healthcare	Financials	IT	Comm	Utilities	Real Estate
Mkt-RF	1.00000	0.10884	-0.17641	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
SMB	0.10884	1.00000	-0.26578	-0.29799	0.25394	0.30683	0.43739	0.06464	0.42801	-0.53977	0.52925	0.18550	0.24169	0.04924
HML	-0.17641	-0.26578	1.00000	0.41847	-0.08912	-0.08892	-0.37708	-0.48999	-0.59235	0.47054	-0.52723	-0.42738	0.08301	0.22374
Energy	0.00000	-0.29799	0.41847	1.00000	0.12149	0.07853	-0.26937	-0.22586	-0.39521	0.36669	-0.28916	-0.20942	0.11513	-0.00057
Materials	0.00000	0.25394	-0.08912	0.12149	1.00000	0.18763	0.12442	-0.04232	0.01964	-0.22344	0.18188	0.07453	0.14172	-0.10054
Industrials	0.00000	0.30683	-0.08892	0.07853	0.18763	1.00000	0.19775	0.03258	0.15058	-0.21622	0.30505	0.29701	0.41480	-0.08378
Cons Disc	0.00000	0.43739	-0.37708	-0.26937	0.12442	0.19775	1.00000	0.32911	0.47690	-0.58398	0.42552	0.22123	0.13102	-0.09229
Cons Stap	0.00000	0.06464	-0.48999	-0.22586	-0.04232	0.03258	0.32911	1.00000	0.48563	-0.21044	0.20268	0.15440	-0.04209	-0.17808
Healthcare	0.00000	0.42801	-0.59235	-0.39521	0.01964	0.15058	0.47690	0.48563	1.00000	-0.51835	0.59415	0.30578	0.15877	-0.20969
Financials	0.00000	-0.53977	0.47054	0.36669	-0.22344	-0.21622	-0.58398	-0.21044	-0.51835	1.00000	-0.46547	-0.12714	-0.15521	0.11344
IT	0.00000	0.52925	-0.52723	-0.28916	0.18188	0.30505	0.42552	0.20268	0.59415	-0.46547	1.00000	0.53067	0.02921	-0.15282
Comm	0.00000	0.18550	-0.42738	-0.20942	0.07453	0.29701	0.22123	0.15440	0.30578	-0.12714	0.53067	1.00000	0.10360	-0.09055
Utilities	0.00000	0.24169	0.08301	0.11513	0.14172	0.41480	0.13102	-0.04209	0.15877	-0.15521	0.02921	0.10360	1.00000	0.00164
Real Estate	0.00000	0.04924	0.22374	-0.00057	-0.10054	-0.08378	-0.09229	-0.17808	-0.20969	0.11344	-0.15282	-0.09055	0.00164	1.00000

Table 10 Correlation of original factors

	Mkt-RF	SMB	HML	Energy	Materials	Industrials	Cons Disc	Cons Stap	Healthcare	Financials	IT	Comm	Utilities	Real Estate
Mkt-RF	1.00000	0.06815	0.06374	-0.00597	-0.00967	-0.01271	-0.00548	0.02767	0.00210	0.00318	0.00145	0.02030	-0.01782	-0.01554
SMB	0.06815	1.00000	-0.18204	0.01706	0.02761	0.03630	0.01566	-0.07902	-0.00598	-0.00909	-0.00414	-0.05799	0.05090	0.04439
HML	0.06374	-0.18204	1.00000	0.01596	0.02582	0.03395	0.01464	-0.07390	-0.00560	-0.00850	-0.00387	-0.05423	0.04761	0.04151
Energy	-0.00597	0.01706	0.01596	1.00000	-0.00242	-0.00318	-0.00137	0.00693	0.00052	0.00080	0.00036	0.00508	-0.00446	-0.00389
Materials	-0.00967	0.02761	0.02582	-0.00242	1.00000	-0.00515	-0.00222	0.01121	0.00085	0.00129	0.00059	0.00822	-0.00722	-0.00630
Industrials	-0.01271	0.03630	0.03395	-0.00318	-0.00515	1.00000	-0.00292	0.01474	0.00112	0.00169	0.00077	0.01081	-0.00949	-0.00828
Cons Disc	-0.00548	0.01566	0.01464	-0.00137	-0.00222	-0.00292	1.00000	0.00636	0.00048	0.00073	0.00033	0.00466	-0.00409	-0.00357
Cons Stap	0.02767	-0.07902	-0.07390	0.00693	0.01121	0.01474	0.00636	1.00000	-0.00243	-0.00369	-0.00168	-0.02354	0.02067	0.01802
Healthcare	0.00210	-0.00598	-0.00560	0.00052	0.00085	0.00112	0.00048	-0.00243	1.00000	-0.00028	-0.00013	-0.00178	0.00156	0.00136
Financials	0.00318	-0.00909	-0.00850	0.00080	0.00129	0.00169	0.00073	-0.00369	-0.00028	1.00000	-0.00019	-0.00271	0.00238	0.00207
IT	0.00145	-0.00414	-0.00387	0.00036	0.00059	0.00077	0.00033	-0.00168	-0.00013	-0.00019	1.00000	-0.00123	0.00108	0.00094
Comm	0.02030	-0.05799	-0.05423	0.00508	0.00822	0.01081	0.00466	-0.02354	-0.00178	-0.00271	-0.00123	1.00000	0.01517	0.01322
Utilities	-0.01782	0.05090	0.04761	-0.00446	-0.00722	-0.00949	-0.00409	0.02067	0.00156	0.00238	0.00108	0.01517	1.00000	-0.01161
Real Estate	-0.01554	0.04439	0.04151	-0.00389	-0.00630	-0.00828	-0.00357	0.01802	0.00136	0.00207	0.00094	0.01322	-0.01161	1.00000

Table 11 Correlation of orthogonalized factors using the GS method

	Mkt-RF	SMB	HML	Energy	Materials	Industrials	Cons Disc	Cons Stap	Healthcare	Financials	IT	Comm	Utilities	Real Estate
Mkt-RF	1.00000	0.05977	0.05637	0.00036	-0.00351	-0.00614	-0.00514	0.01072	0.00230	0.00108	-0.00542	0.00672	-0.00886	-0.00635
SMB	0.05977	1.00000	-0.23122	-0.00148	0.01439	0.02519	0.02110	-0.04395	-0.00942	-0.00442	0.02224	-0.02757	0.03633	0.02606
HML	0.05637	-0.23122	1.00000	-0.00139	0.01357	0.02376	0.01990	-0.04145	-0.00889	-0.00417	0.02098	-0.02600	0.03426	0.02458
Energy	0.00036	-0.00148	-0.00139	1.00000	0.00009	0.00015	0.00013	-0.00026	-0.00006	-0.00003	0.00013	-0.00017	0.00022	0.00016
Materials	-0.00351	0.01439	0.01357	0.00009	1.00000	-0.00148	-0.00124	0.00258	0.00055	0.00026	-0.00131	0.00162	-0.00213	-0.00153
Industrials	-0.00614	0.02519	0.02376	0.00015	-0.00148	1.00000	-0.00217	0.00452	0.00097	0.00045	-0.00229	0.00283	-0.00373	-0.00268
Cons Disc	-0.00514	0.02110	0.01990	0.00013	-0.00124	-0.00217	1.00000	0.00378	0.00081	0.00038	-0.00191	0.00237	-0.00313	-0.00224
Cons Stap	0.01072	-0.04395	-0.04145	-0.00026	0.00258	0.00452	0.00378	1.00000	-0.00169	-0.00079	0.00399	-0.00494	0.00651	0.00467
Healthcare	0.00230	-0.00942	-0.00889	-0.00006	0.00055	0.00097	0.00081	-0.00169	1.00000	-0.00017	0.00085	-0.00106	0.00140	0.00100
Financials	0.00108	-0.00442	-0.00417	-0.00003	0.00026	0.00045	0.00038	-0.00079	-0.00017	1.00000	0.00040	-0.00050	0.00066	0.00047
IT	-0.00542	0.02224	0.02098	0.00013	-0.00131	-0.00229	-0.00191	0.00399	0.00085	0.00040	1.00000	0.00250	-0.00330	-0.00236
Comm	0.00672	-0.02757	-0.02600	-0.00017	0.00162	0.00283	0.00237	-0.00494	-0.00106	-0.00050	0.00250	1.00000	0.00408	0.00293
Utilities	-0.00886	0.03633	0.03426	0.00022	-0.00213	-0.00373	-0.00313	0.00651	0.00140	0.00066	-0.00330	0.00408	1.00000	-0.00386
Real Estate	-0.00635	0.02606	0.02458	0.00016	-0.00153	-0.00268	-0.00224	0.00467	0.00100	0.00047	-0.00236	0.00293	-0.00386	1.00000

Table 12 Correlation of orthogonalized factors using the SW/L method

We then check whether the orthogonalized factors are closely related to the original as we have done for the US market. The table below shows the correlation between original and orthogonalized factors using the two orthogonalization methods.

	Mkt-RF	SMB	HML	Energy	Materials	Industrials	Cons Disc	Cons Stap	Healthcare	Financials	IT	Comm	Utilities	Real Estate
GS	1.00000	0.99917	0.96890	0.88462	0.94193	0.93559	0.85013	0.87528	0.68739	0.68458	0.68058	0.78946	0.84573	0.95240
SW/L	0.99513	0.91110	0.84827	0.92311	0.97070	0.91903	0.87764	0.93467	0.85925	0.85708	0.84903	0.95565	0.96577	0.98613

Table 13 Correlation between original and orthogonalized factors

The SW/L method outperforms the GS method using this overlap measure. Using the GS method, there are four correlations below 0.80 whereas using the SW/L method, all factor correlations are above 0.80.

4.1.2 Comparing the SW/L method over multiple periods

Over the two 5-year periods (2011-2015, 2016-2020), the coefficient matrix generated using the SW/L method is similar. The correlation coefficient between $S_{2011\text{-}2015}$ and $S_{2016\text{-}2020}$ is 0.8625. On average, 57.89% of the variance can be explained by the model.

4.2 The 27-factor model

We also use the same 27-factor model for the Chinese market. The factors are the Fama French three factors plus twenty-four industry group factors based on Global Industry Classification Standard (GICS).

4.2.1 Orthogonalization performance

The tables below show the correlation between factors in the same set.

	Mkt-RF	SMB	HML	1010	1510	2010	2020	2030	2510	2520	2530	2550	3010
Mkt-RF	1.00000	0.37367	0.26729	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
SMB	0.37367	1.00000	0.15744	0.10674	0.01603	-0.01874	-0.02482	0.07177	0.08991	0.18553	0.03801	0.10996	-0.23366
HML	0.26729	0.15744	1.00000	0.22876	0.06446	0.29999	-0.09345	0.16742	0.01031	0.19045	0.07457	-0.21337	-0.07700
1010	0.00000	0.10674	0.22876	1.00000	0.25446	0.05212	-0.07303	-0.17601	-0.13968	-0.17161	-0.03090	-0.32295	-0.27770
1510	0.00000	0.01603	0.06446	0.25446	1.00000	0.28178	-0.03755	0.17462	0.15705	0.03393	0.10902	-0.12670	-0.07250
2010	0.00000	-0.01874	0.29999	0.05212	0.28178	1.00000	0.09483	0.31663	0.10092	0.15821	0.22980	-0.20671	-0.02279
2020	0.00000	-0.02482	-0.09345	-0.07303	-0.03755	0.09483	1.00000	0.14441	-0.04269	-0.01455	0.04714	-0.05936	0.16545
2030	0.00000	0.07177	0.16742	-0.17601	0.17462	0.31663	0.14441	1.00000	0.10174	0.24191	0.21009	0.05402	0.23916
2510	0.00000	0.08991	0.01031	-0.13968	0.15705	0.10092	-0.04269	0.10174	1.00000	0.24334	0.19435	0.14794	0.01710
2520	0.00000	0.18553	0.19045	-0.17161	0.03393	0.15821	-0.01455	0.24191	0.24334	1.00000	0.30083	0.36030	0.11459
2530	0.00000	0.03801	0.07457	-0.03090	0.10902	0.22980	0.04714	0.21009	0.19435	0.30083	1.00000	0.06226	-0.02095
2550	0.00000	0.10996	-0.21337	-0.32295	-0.12670	-0.20671	-0.05936	0.05402	0.14794	0.36030	0.06226	1.00000	0.26536
3010	0.00000	-0.23366	-0.07700	-0.27770	-0.07250	-0.02279	0.16545	0.23916	0.01710	0.11459	-0.02095	0.26536	1.00000
3020	0.00000	-0.37915	-0.01428	-0.04935	-0.16552	-0.02817	0.18656	-0.04611	-0.16054	-0.04914	0.03405	-0.17020	0.17615
3030	0.00000	-0.31862	-0.03961	-0.16309	-0.16463	0.08613	0.19373	0.06340	-0.04042	0.08462	0.07828	-0.10363	0.22859
3510	0.00000	-0.01546	-0.17212	-0.14900	-0.15217	-0.11291	0.04000	-0.19049	-0.23490	-0.10539	-0.08591	0.02075	0.03982
3520	0.00000	-0.18208	-0.19443	-0.19073	-0.17371	-0.12282	-0.06985	-0.08281	-0.23749	-0.21059	-0.16718	-0.02663	0.14154
4010	0.00000	-0.04461	0.66561	-0.10552	-0.10964	0.22687	-0.16228	0.18430	0.11894	0.17336	0.11437	-0.14411	-0.00957
4020	0.00000	-0.09519	0.40644	-0.08212	0.08524	0.12143	-0.06326	0.15013	0.13060	0.07676	0.07789	-0.16748	0.03550
4030	0.00000	-0.11972	0.41337	-0.09399	0.03438	0.28075	0.10064	0.03214	0.02568	0.17657	0.15770	-0.12994	0.01649
4510	0.00000	-0.18334	-0.38961	-0.11961	-0.03524	-0.15626	0.00205	-0.10475	0.07460	-0.17623	0.04160	0.13443	-0.03448
4520	0.00000	0.00218	-0.31174	-0.24431	-0.09115	-0.22037	-0.08148	-0.14371	0.17114	-0.04197	-0.06286	0.14294	-0.06546
4530	0.00000	0.15221	-0.18672	-0.13093	0.08319	-0.05074	-0.06107	0.05379	-0.01333	0.00829	-0.05716	0.19224	-0.02923
5010	0.00000	-0.22021	-0.00830	0.05716	0.00391	0.04988	0.18485	0.06400	-0.05973	-0.09358	0.00075	-0.09179	0.21772
5020	0.00000	-0.14248	-0.19059	-0.08537	-0.06044	-0.16448	0.01445	-0.09176	0.03895	-0.13023	0.20234	0.16228	-0.04648
5510	0.00000	-0.21698	-0.05173	0.00085	-0.11710	-0.10500	0.27818	-0.10716	-0.15084	-0.07449	-0.09647	-0.18465	0.13355
6010	0.00000	0.04008	0.16311	-0.17960	0.06241	0.08112	0.11373	0.06731	0.13328	0.31103	0.23521	0.09089	0.04869

	3020	3030	3510	3520	4010	4020	4030	4510	4520	4530	5010	5020	5510	6010
Mkt-RF	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
SMB	-0.37915	-0.31862	-0.01546	-0.18208	-0.04461	-0.09519	-0.11972	-0.18334	0.00218	0.15221	-0.22021	-0.14248	-0.21698	0.04008
HML	-0.01428	-0.03961	-0.17212	-0.19443	0.66561	0.40644	0.41337	-0.38961	-0.31174	-0.18672	-0.00830	-0.19059	-0.05173	0.16311
1010	-0.04935	-0.16309	-0.14900	-0.19073	-0.10552	-0.08212	-0.09399	-0.11961	-0.24431	-0.13093	0.05716	-0.08537	0.00085	-0.17960
1510	-0.16552	-0.16463	-0.15217	-0.17371	-0.10964	0.08524	0.03438	-0.03524	-0.09115	0.08319	0.00391	-0.06044	-0.11710	0.06241
2010	-0.02817	0.08613	-0.11291	-0.12282	0.22687	0.12143	0.28075	-0.15626	-0.22037	-0.05074	0.04988	-0.16448	-0.10500	0.08112
2020	0.18656	0.19373	0.04000	-0.06985	-0.16228	-0.06326	0.10064	0.00205	-0.08148	-0.06107	0.18485	0.01445	0.27818	0.11373
2030	-0.04611	0.06340	-0.19049	-0.08281	0.18430	0.15013	0.03214	-0.10475	-0.14371	0.05379	0.06400	-0.09176	-0.10716	0.06731
2510	-0.16054	-0.04042	-0.23490	-0.23749	0.11894	0.13060	0.02568	0.07460	0.17114	-0.01333	-0.05973	0.03895	-0.15084	0.13328
2520	-0.04914	0.08462	-0.10539	-0.21059	0.17336	0.07676	0.17657	-0.17623	-0.04197	0.00829	-0.09358	-0.13023	-0.07449	0.31103
2530	0.03405	0.07828	-0.08591	-0.16718	0.11437	0.07789	0.15770	0.04160	-0.06286	-0.05716	0.00075	0.20234	-0.09647	0.23521
2550	-0.17020	-0.10363	0.02075	-0.02663	-0.14411	-0.16748	-0.12994	0.13443	0.14294	0.19224	-0.09179	0.16228	-0.18465	0.09089
3010	0.17615	0.22859	0.03982	0.14154	-0.00957	0.03550	0.01649	-0.03448	-0.06546	-0.02923	0.21772	-0.04648	0.13355	0.04869
3020	1.00000	0.56148	0.15609	0.23221	-0.08721	-0.03985	-0.00137	-0.10154	-0.08535	-0.20324	0.32189	0.08494	0.43118	0.10030
3030	0.56148	1.00000	0.11277	0.27623	-0.05046	-0.06616	0.01886	-0.11010	-0.06158	-0.16749	0.21259	-0.07554	0.30875	0.15771
3510	0.15609	0.11277	1.00000	0.45449	-0.08867	-0.10604	-0.08452	-0.08965	-0.15266	-0.19043	-0.09758	-0.07507	0.10351	0.06876
3520	0.23221	0.27623	0.45449	1.00000	-0.03638	-0.10304	-0.07647	-0.11290	-0.22038	-0.18268	0.11827	-0.17635	0.11339	0.00846
4010	-0.08721	-0.05046	-0.08867	-0.03638	1.00000	0.58675	0.51472	-0.22412	-0.25689	-0.18654	-0.17699	-0.15850	-0.29139	0.11030
4020	-0.03985	-0.06616	-0.10604	-0.10304	0.58675	1.00000	0.33249	0.00319	-0.13228	-0.10484	-0.11465	-0.12109	-0.19720	-0.02445
4030	-0.00137	0.01886	-0.08452	-0.07647	0.51472	0.33249	1.00000	-0.15486	-0.26336	-0.18332	-0.02253	-0.14467	0.00959	0.19299
4510	-0.10154	-0.11010	-0.08965	-0.11290	-0.22412	0.00319	-0.15486	1.00000	0.33535	0.17150	0.03445	0.23961	-0.06340	-0.10115
4520	-0.08535	-0.06158	-0.15266	-0.22038	-0.25689	-0.13228	-0.26336	0.33535	1.00000	0.41650	-0.08911	0.05306	-0.05503	-0.01829
4530	-0.20324	-0.16749	-0.19043	-0.18268	-0.18654	-0.10484	-0.18332	0.17150	0.41650	1.00000	-0.19977	0.02428	-0.16995	-0.07809
5010	0.32189	0.21259	-0.09758	0.11827	-0.17699	-0.11465	-0.02253	0.03445	-0.08911	-0.19977	1.00000	0.00527	0.33872	0.05139
5020	0.08494	-0.07554	-0.07507	-0.17635	-0.15850	-0.12109	-0.14467	0.23961	0.05306	0.02428	0.00527	1.00000	-0.03081	-0.04089
5510	0.43118	0.30875	0.10351	0.11339	-0.29139	-0.19720	0.00959	-0.06340	-0.05503	-0.16995	0.33872	-0.03081	1.00000	0.30151
6010	0.10030	0.15771	0.06876	0.00846	0.11030	-0.02445	0.19299	-0.10115	-0.01829	-0.07809	0.05139	-0.04089	0.30151	1.00000

Table 14 Correlation of original factors

	Mkt-RF	SMB	HML	1010	1510	2010	2020	2030	2510	2520	2530	2550	3010
Mkt-RF	1.00000	0.06815	0.06374	-0.00597	-0.00967	-0.00759	-0.01006	-0.01497	-0.00975	-0.00811	-0.00013	0.01747	-0.00152
SMB	0.06815	1.00000	-0.18204	0.01706	0.02761	0.02169	0.02874	0.04274	0.02783	0.02316	0.00038	-0.04991	0.00436
HML	0.06374	-0.18204	1.00000	0.01596	0.02582	0.02028	0.02688	0.03998	0.02603	0.02166	0.00036	-0.04668	0.00407
1010	-0.00597	0.01706	0.01596	1.00000	-0.00242	-0.00190	-0.00252	-0.00375	-0.00244	-0.00203	-0.00003	0.00438	-0.00038
1510	-0.00967	0.02761	0.02582	-0.00242	1.00000	-0.00308	-0.00408	-0.00606	-0.00395	-0.00329	-0.00005	0.00708	-0.00062
2010	-0.00759	0.02169	0.02028	-0.00190	-0.00308	1.00000	-0.00320	-0.00476	-0.00310	-0.00258	-0.00004	0.00556	-0.00049
2020	-0.01006	0.02874	0.02688	-0.00252	-0.00408	-0.00320	1.00000	-0.00631	-0.00411	-0.00342	-0.00006	0.00737	-0.00064
2030	-0.01497	0.04274	0.03998	-0.00375	-0.00606	-0.00476	-0.00631	1.00000	-0.00611	-0.00509	-0.00008	0.01096	-0.00096
2510	-0.00975	0.02783	0.02603	-0.00244	-0.00395	-0.00310	-0.00411	-0.00611	1.00000	-0.00331	-0.00005	0.00714	-0.00062
2520	-0.00811	0.02316	0.02166	-0.00203	-0.00329	-0.00258	-0.00342	-0.00509	-0.00331	1.00000	-0.00005	0.00594	-0.00052
2530	-0.00013	0.00038	0.00036	-0.00003	-0.00005	-0.00004	-0.00006	-0.00008	-0.00005	-0.00005	1.00000	0.00010	-0.00001
2550	0.01747	-0.04991	-0.04668	0.00438	0.00708	0.00556	0.00737	0.01096	0.00714	0.00594	0.00010	1.00000	0.00112
3010	-0.00152	0.00436	0.00407	-0.00038	-0.00062	-0.00049	-0.00064	-0.00096	-0.00062	-0.00052	-0.00001	0.00112	1.00000
3020	0.02454	-0.07010	-0.06556	0.00615	0.00994	0.00781	0.01035	0.01539	0.01002	0.00834	0.00014	-0.01798	0.00157
3030	0.00018	-0.00051	-0.00047	0.00004	0.00007	0.00006	0.00007	0.00011	0.00007	0.00006	0.00000	-0.00013	0.00001
3510	-0.01113	0.03179	0.02973	-0.00279	-0.00451	-0.00354	-0.00469	-0.00698	-0.00455	-0.00378	-0.00006	0.00815	-0.00071
3520	0.00804	-0.02297	-0.02148	0.00201	0.00326	0.00256	0.00339	0.00504	0.00328	0.00273	0.00004	-0.00589	0.00051
4010	-0.00473	0.01352	0.01265	-0.00119	-0.00192	-0.00151	-0.00200	-0.00297	-0.00193	-0.00161	-0.00003	0.00347	-0.00030
4020	0.00388	-0.01108	-0.01037	0.00097	0.00157	0.00123	0.00164	0.00243	0.00158	0.00132	0.00002	-0.00284	0.00025
4030	0.01261	-0.03601	-0.03368	0.00316	0.00511	0.00401	0.00532	0.00791	0.00515	0.00429	0.00007	-0.00923	0.00081
4510	0.00942	-0.02690	-0.02515	0.00236	0.00381	0.00300	0.00397	0.00591	0.00385	0.00320	0.00005	-0.00690	0.00060
4520	0.00134	-0.00384	-0.00359	0.00034	0.00054	0.00043	0.00057	0.00084	0.00055	0.00046	0.00001	-0.00098	0.00009
4530	0.00207	-0.00592	-0.00553	0.00052	0.00084	0.00066	0.00087	0.00130	0.00085	0.00070	0.00001	-0.00152	0.00013
5010	0.00107	-0.00307	-0.00287	0.00027	0.00044	0.00034	0.00045	0.00067	0.00044	0.00037	0.00001	-0.00079	0.00007
5020	0.02054	-0.05866	-0.05486	0.00514	0.00832	0.00653	0.00866	0.01288	0.00839	0.00698	0.00011	-0.01504	0.00131
5510	-0.01242	0.03547	0.03317	-0.00311	-0.00503	-0.00395	-0.00524	-0.00779	-0.00507	-0.00422	-0.00007	0.00909	-0.00079
6010	-0.01199	0.03426	0.03204	-0.00300	-0.00486	-0.00382	-0.00506	-0.00752	-0.00490	-0.00408	-0.00007	0.00878	-0.00077

	3020	3030	3510	3520	4010	4020	4030	4510	4520	4530	5010	5020	5510	6010
Mkt-RF	0.02454	0.00018	-0.01113	0.00804	-0.00473	0.00388	0.01261	0.00942	0.00134	0.00207	0.00107	0.02054	-0.01242	-0.01199
SMB	-0.07010	-0.00051	0.03179	-0.02297	0.01352	-0.01108	-0.03601	-0.02690	-0.00384	-0.00592	-0.00307	-0.05866	0.03547	0.03426
HML	-0.06556	-0.00047	0.02973	-0.02148	0.01265	-0.01037	-0.03368	-0.02515	-0.00359	-0.00553	-0.00287	-0.05486	0.03317	0.03204
1010	0.00615	0.00004	-0.00279	0.00201	-0.00119	0.00097	0.00316	0.00236	0.00034	0.00052	0.00027	0.00514	-0.00311	-0.00300
1510	0.00994	0.00007	-0.00451	0.00326	-0.00192	0.00157	0.00511	0.00381	0.00054	0.00084	0.00044	0.00832	-0.00503	-0.00486
2010	0.00781	0.00006	-0.00354	0.00256	-0.00151	0.00123	0.00401	0.00300	0.00043	0.00066	0.00034	0.00653	-0.00395	-0.00382
2020	0.01035	0.00007	-0.00469	0.00339	-0.00200	0.00164	0.00532	0.00397	0.00057	0.00087	0.00045	0.00866	-0.00524	-0.00506
2030	0.01539	0.00011	-0.00698	0.00504	-0.00297	0.00243	0.00791	0.00591	0.00084	0.00130	0.00067	0.01288	-0.00779	-0.00752
2510	0.01002	0.00007	-0.00455	0.00328	-0.00193	0.00158	0.00515	0.00385	0.00055	0.00085	0.00044	0.00839	-0.00507	-0.00490
2520	0.00834	0.00006	-0.00378	0.00273	-0.00161	0.00132	0.00429	0.00320	0.00046	0.00070	0.00037	0.00698	-0.00422	-0.00408
2530	0.00014	0.00000	-0.00006	0.00004	-0.00003	0.00002	0.00007	0.00005	0.00001	0.00001	0.00001	0.00011	-0.00007	-0.00007
2550	-0.01798	-0.00013	0.00815	-0.00589	0.00347	-0.00284	-0.00923	-0.00690	-0.00098	-0.00152	-0.00079	-0.01504	0.00909	0.00878
3010	0.00157	0.00001	-0.00071	0.00051	-0.00030	0.00025	0.00081	0.00060	0.00009	0.00013	0.00007	0.00131	-0.00079	-0.00077
3020	1.00000	-0.00018	0.01145	-0.00827	0.00487	-0.00399	-0.01297	-0.00969	-0.00138	-0.00213	-0.00111	-0.02113	0.01277	0.01234
3030	-0.00018	1.00000	0.00008	-0.00006	0.00004	-0.00003	-0.00009	-0.00007	-0.00001	-0.00002	-0.00001	-0.00015	0.00009	0.00009
3510	0.01145	0.00008	1.00000	0.00375	-0.00221	0.00181	0.00588	0.00439	0.00063	0.00097	0.00050	0.00958	-0.00579	-0.00559
3520	-0.00827	-0.00006	0.00375	1.00000	0.00160	-0.00131	-0.00425	-0.00317	-0.00045	-0.00070	-0.00036	-0.00692	0.00418	0.00404
4010	0.00487	0.00004	-0.00221	0.00160	1.00000	0.00077	0.00250	0.00187	0.00027	0.00041	0.00021	0.00407	-0.00246	-0.00238
4020	-0.00399	-0.00003	0.00181	-0.00131	0.00077	1.00000	-0.00205	-0.00153	-0.00022	-0.00034	-0.00017	-0.00334	0.00202	0.00195
4030	-0.01297	-0.00009	0.00588	-0.00425	0.00250	-0.00205	1.00000	-0.00498	-0.00071	-0.00109	-0.00057	-0.01085	0.00656	0.00634
4510	-0.00969	-0.00007	0.00439	-0.00317	0.00187	-0.00153	-0.00498	1.00000	-0.00053	-0.00082	-0.00042	-0.00811	0.00490	0.00473
4520	-0.00138	-0.00001	0.00063	-0.00045	0.00027	-0.00022	-0.00071	-0.00053	1.00000	-0.00012	-0.00006	-0.00116	0.00070	0.00068
4530	-0.00213	-0.00002	0.00097	-0.00070	0.00041	-0.00034	-0.00109	-0.00082	-0.00012	1.00000	-0.00009	-0.00178	0.00108	0.00104
5010	-0.00111	-0.00001	0.00050	-0.00036	0.00021	-0.00017	-0.00057	-0.00042	-0.00006	-0.00009	1.00000	-0.00092	0.00056	0.00054
5020	-0.02113	-0.00015	0.00958	-0.00692	0.00407	-0.00334	-0.01085	-0.00811	-0.00116	-0.00178	-0.00092	1.00000	0.01069	0.01032
5510	0.01277	0.00009	-0.00579	0.00418	-0.00246	0.00202	0.00656	0.00490	0.00070	0.00108	0.00056	0.01069	1.00000	-0.00624
6010	0.01234	0.00009	-0.00559	0.00404	-0.00238	0.00195	0.00634	0.00473	0.00068	0.00104	0.00054	0.01032	-0.00624	1.00000

Table 15 Correlation of orthogonalized factors using the GS method

	Mkt-RF	SMB	HML	1010	1510	2010	2020	2030	2510	2520	2530	2550	3010
Mkt-RF	1.00000	0.06214	0.05734	-0.00057	-0.00513	-0.00039	-0.00334	-0.00680	-0.00428	-0.00397	-0.00020	0.00396	-0.00261
SMB	0.06214	1.00000	-0.24849	0.00246	0.02224	0.00167	0.01449	0.02946	0.01854	0.01719	0.00088	-0.01716	0.01130
HML	0.05734	-0.24849	1.00000	0.00227	0.02053	0.00154	0.01337	0.02719	0.01711	0.01586	0.00081	-0.01583	0.01043
1010	-0.00057	0.00246	0.00227	1.00000	-0.00020	-0.00002	-0.00013	-0.00027	-0.00017	-0.00016	-0.00001	0.00016	-0.00010
1510	-0.00513	0.02224	0.02053	-0.00020	1.00000	-0.00014	-0.00120	-0.00243	-0.00153	-0.00142	-0.00007	0.00142	-0.00093
2010	-0.00039	0.00167	0.00154	-0.00002	-0.00014	1.00000	-0.00009	-0.00018	-0.00012	-0.00011	-0.00001	0.00011	-0.00007
2020	-0.00334	0.01449	0.01337	-0.00013	-0.00120	-0.00009	1.00000	-0.00159	-0.00100	-0.00092	-0.00005	0.00092	-0.00061
2030	-0.00680	0.02946	0.02719	-0.00027	-0.00243	-0.00018	-0.00159	1.00000	-0.00203	-0.00188	-0.00010	0.00188	-0.00124
2510	-0.00428	0.01854	0.01711	-0.00017	-0.00153	-0.00012	-0.00100	-0.00203	1.00000	-0.00118	-0.00006	0.00118	-0.00078
2520	-0.00397	0.01719	0.01586	-0.00016	-0.00142	-0.00011	-0.00092	-0.00188	-0.00118	1.00000	-0.00006	0.00110	-0.00072
2530	-0.00020	0.00088	0.00081	-0.00001	-0.00007	-0.00001	-0.00005	-0.00010	-0.00006	-0.00006	1.00000	0.00006	-0.00004
2550	0.00396	-0.01716	-0.01583	0.00016	0.00142	0.00011	0.00092	0.00188	0.00118	0.00110	0.00006	1.00000	0.00072
3010	-0.00261	0.01130	0.01043	-0.00010	-0.00093	-0.00007	-0.00061	-0.00124	-0.00078	-0.00072	-0.00004	0.00072	1.00000
3020	0.01183	-0.05126	-0.04730	0.00047	0.00423	0.00032	0.00276	0.00561	0.00353	0.00327	0.00017	-0.00327	0.00215
3030	0.00025	-0.00107	-0.00099	0.00001	0.00009	0.00001	0.00006	0.00012	0.00007	0.00007	0.00000	-0.00007	0.00004
3510	-0.00606	0.02625	0.02422	-0.00024	-0.00217	-0.00016	-0.00141	-0.00287	-0.00181	-0.00168	-0.00009	0.00167	-0.00110
3520	0.00451	-0.01953	-0.01802	0.00018	0.00161	0.00012	0.00105	0.00214	0.00134	0.00125	0.00006	-0.00124	0.00082
4010	-0.00124	0.00539	0.00497	-0.00005	-0.00045	-0.00003	-0.00029	-0.00059	-0.00037	-0.00034	-0.00002	0.00034	-0.00023
4020	-0.00023	0.00098	0.00090	-0.00001	-0.00008	-0.00001	-0.00005	-0.00011	-0.00007	-0.00006	0.00000	0.00006	-0.00004
4030	0.00478	-0.02070	-0.01910	0.00019	0.00171	0.00013	0.00111	0.00227	0.00143	0.00132	0.00007	-0.00132	0.00087
4510	-0.00069	0.00298	0.00275	-0.00003	-0.00025	-0.00002	-0.00016	-0.00033	-0.00021	-0.00019	-0.00001	0.00019	-0.00013
4520	-0.00140	0.00606	0.00559	-0.00006	-0.00050	-0.00004	-0.00033	-0.00066	-0.00042	-0.00039	-0.00002	0.00039	-0.00025
4530	-0.00169	0.00731	0.00674	-0.00007	-0.00060	-0.00005	-0.00039	-0.00080	-0.00050	-0.00047	-0.00002	0.00047	-0.00031
5010	0.00003	-0.00015	-0.00014	0.00000	0.00001	0.00000	0.00001	0.00002	0.00001	0.00001	0.00000	-0.00001	0.00001
5020	0.00597	-0.02587	-0.02387	0.00024	0.00214	0.00016	0.00139	0.00283	0.00178	0.00165	0.00008	-0.00165	0.00109
5510	-0.00725	0.03143	0.02900	-0.00029	-0.00260	-0.00020	-0.00169	-0.00344	-0.00216	-0.00201	-0.00010	0.00200	-0.00132
6010	-0.00600	0.02599	0.02398	-0.00024	-0.00215	-0.00016	-0.00140	-0.00284	-0.00179	-0.00166	-0.00008	0.00166	-0.00109

	3020	3030	3510	3520	4010	4020	4030	4510	4520	4530	5010	5020	5510	6010
Mkt-RF	0.01183	0.00025	-0.00606	0.00451	-0.00124	-0.00023	0.00478	-0.00069	-0.00140	-0.00169	0.00003	0.00597	-0.00725	-0.00600
SMB	-0.05126	-0.00107	0.02625	-0.01953	0.00539	0.00098	-0.02070	0.00298	0.00606	0.00731	-0.00015	-0.02587	0.03143	0.02599
HML	-0.04730	-0.00099	0.02422	-0.01802	0.00497	0.00090	-0.01910	0.00275	0.00559	0.00674	-0.00014	-0.02387	0.02900	0.02398
1010	0.00047	0.00001	-0.00024	0.00018	-0.00005	-0.00001	0.00019	-0.00003	-0.00006	-0.00007	0.00000	0.00024	-0.00029	-0.00024
1510	0.00423	0.00009	-0.00217	0.00161	-0.00045	-0.00008	0.00171	-0.00025	-0.00050	-0.00060	0.00001	0.00214	-0.00260	-0.00215
2010	0.00032	0.00001	-0.00016	0.00012	-0.00003	-0.00001	0.00013	-0.00002	-0.00004	-0.00005	0.00000	0.00016	-0.00020	-0.00016
2020	0.00276	0.00006	-0.00141	0.00105	-0.00029	-0.00005	0.00111	-0.00016	-0.00033	-0.00039	0.00001	0.00139	-0.00169	-0.00140
2030	0.00561	0.00012	-0.00287	0.00214	-0.00059	-0.00011	0.00227	-0.00033	-0.00066	-0.00080	0.00002	0.00283	-0.00344	-0.00284
2510	0.00353	0.00007	-0.00181	0.00134	-0.00037	-0.00007	0.00143	-0.00021	-0.00042	-0.00050	0.00001	0.00178	-0.00216	-0.00179
2520	0.00327	0.00007	-0.00168	0.00125	-0.00034	-0.00006	0.00132	-0.00019	-0.00039	-0.00047	0.00001	0.00165	-0.00201	-0.00166
2530	0.00017	0.00000	-0.00009	0.00006	-0.00002	0.00000	0.00007	-0.00001	-0.00002	-0.00002	0.00000	0.00008	-0.00010	-0.00008
2550	-0.00327	-0.00007	0.00167	-0.00124	0.00034	0.00006	-0.00132	0.00019	0.00039	0.00047	-0.00001	-0.00165	0.00200	0.00166
3010	0.00215	0.00004	-0.00110	0.00082	-0.00023	-0.00004	0.00087	-0.00013	-0.00025	-0.00031	0.00001	0.00109	-0.00132	-0.00109
3020	1.00000	-0.00020	0.00500	-0.00372	0.00103	0.00019	-0.00394	0.00057	0.00115	0.00139	-0.00003	-0.00492	0.00598	0.00495
3030	-0.00020	1.00000	0.00010	-0.00008	0.00002	0.00000	-0.00008	0.00001	0.00002	0.00003	0.00000	-0.00010	0.00012	0.00010
3510	0.00500	0.00010	1.00000	0.00190	-0.00053	-0.00010	0.00202	-0.00029	-0.00059	-0.00071	0.00001	0.00252	-0.00306	-0.00253
3520	-0.00372	-0.00008	0.00190	1.00000	0.00039	0.00007	-0.00150	0.00022	0.00044	0.00053	-0.00001	-0.00188	0.00228	0.00188
4010	0.00103	0.00002	-0.00053	0.00039	1.00000	-0.00002	0.00041	-0.00006	-0.00012	-0.00015	0.00000	0.00052	-0.00063	-0.00052
4020	0.00019	0.00000	-0.00010	0.00007	-0.00002	1.00000	0.00008	-0.00001	-0.00002	-0.00003	0.00000	0.00009	-0.00011	-0.00009
4030	-0.00394	-0.00008	0.00202	-0.00150	0.00041	0.00008	1.00000	0.00023	0.00047	0.00056	-0.00001	-0.00199	0.00242	0.00200
4510	0.00057	0.00001	-0.00029	0.00022	-0.00006	-0.00001	0.00023	1.00000	-0.00007	-0.00008	0.00000	0.00029	-0.00035	-0.00029
4520	0.00115	0.00002	-0.00059	0.00044	-0.00012	-0.00002	0.00047	-0.00007	1.00000	-0.00016	0.00000	0.00058	-0.00071	-0.00058
4530	0.00139	0.00003	-0.00071	0.00053	-0.00015	-0.00003	0.00056	-0.00008	-0.00016	1.00000	0.00000	0.00070	-0.00085	-0.00071
5010	-0.00003	0.00000	0.00001	-0.00001	0.00000	0.00000	-0.00001	0.00000	0.00000	0.00000	1.00000	-0.00001	0.00002	0.00001
5020	-0.00492	-0.00010	0.00252	-0.00188	0.00052	0.00009	-0.00199	0.00029	0.00058	0.00070	-0.00001	1.00000	0.00302	0.00250
5510	0.00598	0.00012	-0.00306	0.00228	-0.00063	-0.00011	0.00242	-0.00035	-0.00071	-0.00085	0.00002	0.00302	1.00000	-0.00303
6010	0.00495	0.00010	-0.00253	0.00188	-0.00052	-0.00009	0.00200	-0.00029	-0.00058	-0.00071	0.00001	0.00250	-0.00303	1.00000

Table 16 Correlation of orthogonalized factors using the SW/L method

We check the overlap measure describing how close the orthogonalized factors are related to the original. The table below shows the correlation between original and orthogonalized factors using the GS and SW/L methods.

	Mkt-RF	SMB	HML	1010	1510	2010	2020	2030	2510	2520	2530	2550	3010	3020
GS	1.00000	0.99917	0.96890	0.88462	0.94193	0.92812	0.81933	0.94563	0.95148	0.85193	0.93450	0.84115	0.84541	0.83123
SW/L	0.99481	0.84471	0.79151	0.89465	0.94157	0.89834	0.89734	0.93960	0.95572	0.88960	0.94073	0.87018	0.93033	0.88421
	3030	3510	3520	4010	4020	4030	4510	4520	4530	5010	5020	5510	6010	
GS	0.87049	0.71264	0.56987	0.66380	0.86097	0.85398	0.69555	0.60154	0.63601	0.80988	0.65882	0.78771	0.82170	
SW/L	0.92711	0.80204	0.77215	0.82239	0.96533	0.93479	0.81438	0.74029	0.86859	0.97006	0.87768	0.92981	0.95417	

Table 17 Correlation between original and orthogonalized factors

Using the GS method, out of the twenty-seven correlations between new and original industry group factors, eight correlations are below 0.80 with the lowest value being 0.57.

Using the SW/L method, only three of the factor correlations are below 0.80 with the lowest value being 0.74 for the Banks industry group (4010). Similar to the US market, the advantage of SW/L is generally more significant when the number of factors increases.

4.2.2 Comparing the SW/L method over multiple periods

We also investigate the stability of the coefficient matrix relating old factors to new factors by considering the two 5-year periods (2011-2015, 2016-2020). The correlation coefficient between $S_{2011\text{-}2015}$ and $S_{2016\text{-}2020}$ is 0.7805 on average. The stability is not as high as in the US market. This might be due to the smaller universe we use to construct the risk model for the Chinese market. On average 67.29% of the variance can be explained by the model. This is higher than what can be achieved for the corresponding risk model in the US market.

5. Conclusion

The results we obtained from the above four models demonstrate that the SW/L method gives a significantly higher overlap between the orthogonalized factors and the original factors than what can be obtained using the GS method, especially when more factors are included in the risk model. The coefficient matrix relating old factors to new factors is also shown to be quite stable; this allows for the construction of a robust risk model based on orthogonalized factors which can retain the economic intuition contained in the original factors. Our 27-factor risk model can be used as a baseline model, as it can explain a significant percentage of variance in the stock returns. Our pilot study of this model points to the feasibility of the SW/L method for constructing a useful risk model with linearly independent factors. We plan to extend our study to construct a more complete risk model by using additional style factors and a finer level of the industry exposure (namely exposures to 69 GICS industries) and using a bigger universe.

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