# Volatility and Expected Option Returns

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**Abstract**

We analyze the relation between expected option returns and the volatility of the underlying securities. In the Black-Scholes-Merton and stochastic volatility models, the expected return from holding a call (put) option is a decreasing (increasing) function of the volatility of the underlying. These predictions are strongly supported by the data. In the cross-section of stock option returns, returns on call (put) option portfolios decrease (increase) with underlying stock volatility. This strong negative (positive) relation between call (put) option returns and volatility is not due to cross-sectional variation in expected stock returns. It holds in various option samples with different maturities and moneyness, and it is robust to alternative measures of underlying volatility and different weighting methods. Time-series evidence also supports the predictions from option pricing theory: Future returns on S&P 500 index call (put) options are negatively (positively) related to S&P 500 index volatility.

JEL Classification: G12

Keywords: expected option returns; volatility; cross-section of option returns.

# 1 Introduction

Coval and Shumway (2001) firmly integrate the study of expected option returns into mainstream asset pricing theory. They find that index option data confirm the theoretically expected relation between moneyness and expected returns on puts and calls. More recently, the empirical literature on the cross-section of equity option returns has been expanding rapidly, along with increasing liquidity and data availability. For example, Boyer and Vorkink (2014) investigate the relation between skewness and option returns. Goodman, Neamtiu, and Zhang (2013) find that fundamental accounting information is related to future option returns. A related literature documents the impact of inventory and order flow on option returns (see Muravyev, 2015 and the references therein).

Several papers control for volatility when investigating determinants of the cross-section of option returns, but the use of volatility as a control variable is usually motivated by discussing the relation between volatility and option *prices*. One argument considers the effect of an unexpected (future) change in volatility. This increases the future option price and therefore returns. From the perspective of asset pricing theory, which emphasizes the relation between ex ante risk and expected return, this argument is incomplete. An alternative argument considers a contemporaneous increase or shock in volatility. This increases the current price of the option, which leads to the hypothesis that volatility and returns are negatively related. This argument applies to a transitory shock to volatility, which ignores that changes in the volatility level may affect the future option payoff. We conclude that when discussing volatility and option returns, several arguments are used in the empirical literature that may refer to current or future volatility, to anticipated or unanticipated changes in volatility, and to the impact of volatility on current or future prices.

This paper attempts to contribute to this literature by explicitly considering the ex ante relation between volatility and expected option returns. This integrates the analysis of volatility as a determinant of expected option returns into mainstream asset pricing theory, following Coval and Shumway's (2001) analysis of moneyness. We analytically study the relation between volatility and discrete holding period returns, and empirically investigate this theoretical prediction. Building on the work of Rubinstein (1984) and Broadie, Chernov, and Johannes (2009), we first use analytical expressions for expected holding period option returns in the context of the Black-Scholes-Merton framework. The expected return on holding a call option is a decreasing function of the underlying volatility, while the expected return on holding a put option is an increasing function of the underlying volatility.

Our results can easily be understood in terms of leverage, consistent with the intuition in

Coval and Shumway (2001), who analyze index call and put returns as a function of the leverage due to moneyness. The leverage embedded in an option is a function of moneyness, maturity, and volatility. Figure 1 plots expected returns as a function of volatility for an ATM option with one month maturity. Expected call returns are positive and expected put returns are negative, following the arguments in Coval and Shumway (2001). For both calls and puts, the absolute value of returns is higher for the low-volatility options. This reflects leverage: low volatility options are cheaper and therefore constitute a more leveraged position.

We provide several extensions of the benchmark analysis. The empirical shortcomings of the Black-Scholes-Merton model are well-documented, and we therefore investigate if realistic extensions of the Black-Scholes-Merton model lead to different theoretical predictions. We use realistic parameterizations of the Heston (1993) model to show that if volatility is time-varying and if the innovations to volatility and returns are correlated, similar predictions obtain.

We provide cross-sectional and time-series tests of this theoretical relation between stock volatility and expected option returns. Using the cross-section of stock option returns for 1996-2013, we document that call (put) option portfolio returns exhibit a strong negative (positive) relation with underlying stock volatilities. Sorting available one-month at-the-money options into quintiles, we find a statistically significant difference of $-13.8\%$ ($7.1\%$) per month between the average returns of the call (put) option portfolio with the highest underlying stock volatilities and the call (put) portfolio with the lowest underlying volatilities. We demonstrate that these findings are not driven by cross-sectional variation in expected stock returns. Our results are robust to using different option maturities and moneyness, alternative measures of underlying volatility and portfolio weighting methods, and relevant control variables.

We also provide time-series evidence. We find that index call (put) options tend to have lower (higher) returns in the month following high volatility periods. The findings are robust to different index volatility proxies and are not driven by illiquid option contracts. The time-series results complement our cross-sectional findings and provide empirical support for our theoretical predictions.

To the best of our knowledge the cross-sectional relation between option returns and volatility has not been documented in the empirical asset pricing literature, but some existing studies contain related results. Galai and Masulis (1976) and Johnson (2004) study very different empirical questions related to capital structure and earnings forecasts respectively, but their analytical results are related, exploiting the relation between volatility and *instantaneous* expected option returns. Galai and Masulis (1976) argue that, under the joint assumption of the CAPM and the Black-Scholes-Merton model, the expected instantaneous rate of return on firm equity, which is a call option on firm value, decreases with the variance of the rate of return on firm value under certain (realistic) additional restrictions. Johnson (2004) points out that in a levered firm, the instantaneous expected equity return decreases as a function of idiosyncratic asset risk. He uses this insight to explain the puzzling negative relation between stock returns and the dispersion of analysts' earnings forecasts. We show that in a Black-Scholes-Merton setup, the negative (positive) relation between expected call (put) option return and underlying volatility can be generalized to empirically observable holding periods, and we provide empirical evidence consistent with these theoretical predictions. It is well known (see for instance Broadie, Chernov, and Johannes, 2009) that results for instantaneous returns may not generalize to empirically observable holding periods, because the option price is a convex function of the price of underlying security. Our focus on holding period returns instead of instantaneous returns facilitates the interpretation of the empirical results. It also has certain analytical advantages, which become apparent when we analyze stochastic volatility models.

In other related work, Lyle (2014) explores the implications of the negative relation between expected call option returns and underlying volatility to study the relation between information quality and future option and stock returns. Broadie, Chernov, and Johannes (2009) use simulations to show that expected put option returns increase with underlying volatility.

Finally, recent empirical work on equity options has documented several interesting patterns in the cross-section of option returns that are related to the volatility of the underlying securities. Goyal and Saretto (2009) show that straddle returns and delta-hedged call option returns increase as a function of the volatility risk premium, the difference between historical volatility and implied volatility. Vasquez (2012) reports a positive relation between the slope of the implied volatility term structure and future option returns. Cao and Han (2013) document a negative relation between the underlying stock's idiosyncratic volatility and delta-hedged equity option returns. Duarte and Jones (2007) analyze the relation between delta-hedged equity option returns and volatility betas. These studies all focus on volatility, but they analyze its impact on delta-hedged returns and straddles. Under the null hypothesis that the Black-Scholes-Merton model is correctly specified, volatility should not affect delta-hedged returns in these studies, and therefore the main focus of these papers is by definition on the sources of model misspecification. The objective of our paper is instead to analyze the theoretical and empirical relation between volatility and raw option returns, and to integrate this analysis into the mainstream asset pricing literature. Volatility is one of the main determinants of option prices and returns. Given that the theoretical predicted relations are validated by the data, our work suggests that empirical work on option returns may want to control for the effect of volatility when identifying other determinants of option returns.

The paper proceeds as follows. Section 2 provides the analytical results on the relation between expected option returns and underlying stock volatility in the Black-Scholes-Merton model. Section 3 discusses the data. Section 4 presents our main empirical results, using data on the cross-section of stock option returns. Section 4 also presents results on straddles and investigates expected returns in a stochastic volatility model. Section 5 performs an extensive set of robustness checks. Section 6 discusses several extensions as well as related results. Section 7 presents time-series tests using index options, and Section 8 concludes the paper.

## 2 Volatility and Expected Option Returns

In this section, we derive the analytical results on the relation between option returns and the volatility of the underlying security. We first derive these results in the context of the Black-Scholes-Merton model (Black and Scholes, 1973; Merton, 1973), even though it is well known that the Black-Scholes-Merton model has some empirical shortcomings. Most importantly, more accurate valuation of options is possible by accommodating stochastic volatility as well as jumps in returns and volatility.[1] However, the Black-Scholes-Merton model has the important advantage of analytical tractability, and we therefore use it to derive a benchmark set of theoretical results. In Section 4.4, we investigate if these results continue to hold if other, more realistic, processes are assumed for the underlying securities.

Much of the literature on option returns uses expected instantaneous option returns. In the Black-Scholes-Merton model, consider the following notation for the geometric Brownian motion dynamic of the underlying asset:

$$\frac{dS_t}{S_t} = \mu dt + \sigma dB_t \tag{2.1}$$

where $S_t$ is the price of underlying asset at time $t$, $\sigma$ is the volatility parameter, and $\mu$ is the drift or the expected return of the underlying asset. It can be shown that in this model, the expected instantaneous option return is linear in the expected instantaneous return on the underlying asset:

$$E(\frac{dO_t}{O_t}) = rdt + \frac{\partial O_t}{\partial S_t}\frac{S_t}{O_t}(\mu - r)dt \tag{2.2}$$

where $O_t$ is the price of the European option, and $r$ is the risk-free rate. This expression provides some valuable intuition regarding the determinants of expected option returns. The expected option return depends on $\frac{\partial O_t}{\partial S_t}\frac{S_t}{O_t}$, which reflects the leverage embedded in the option. The leverage itself is a function of moneyness, maturity, and the volatility of the underlying security.

[1] For studies of option pricing with stochastic volatility and jumps, see for instance Bates (1996), Bakshi, Cao, and Chen (1997), Chernov and Ghysels (2000), Eraker (2004), Jones (2003), Pan (2002), and Broadie, Chernov, and Johannes (2007).

Equation (2.2) provides valuable intuition on option returns, but it has some important drawbacks and limitations. Some of these drawbacks follow from the fact that for empirically observable holding periods, the linear relation between the option returns and the underlying asset returns may not hold because the option price is a convex function of the price of underlying asset. For more complex stochastic volatility models, these drawbacks are more severe, see Broadie, Chernov, and Johannes (2009). We analyze stochastic volatility models in Section 4.4.

These drawbacks also surface when analyzing the relation between volatility and expected option returns. We can use (2.2) to compute the derivative of expected returns with respect to volatility $\sigma$. Galai and Masulis (1976), in their analysis of the optionality of leveraged equity, characterize sufficient conditions for this derivative to be negative for a call option when the underlying dynamic is given by (2.1). Johnson (2004), using a similar setup, notes that the derivative is always negative for call options. Because these statements are somewhat contradictory, and also because this result does not seem to be sufficiently appreciated in the literature, we include it in Appendix A.

We now investigate if this result holds for empirically observable holding periods, where we have to account for the fact that the option price is a convex function of the price of underlying security. We analyze the impact of underlying volatility on expected option returns by building on the work of Rubinstein (1984) and Broadie, Chernov, and Johannes (2009), who point out that expected option returns can be computed analytically within models that allow for analytic expressions for option prices. For our benchmark results, we rely on the classical Black-Scholes-Merton option pricing model to obtain an analytical expression for the expected return of holding an option to maturity. We then compute the first derivative of the expected option return with respect to the volatility of the underlying security. We show that the expected return for holding a call option to maturity is a decreasing function of the underlying volatility, while the expected return for holding a put option to maturity is an increasing function of underlying volatility.

Denote the time $t$ prices of European call and put options with strike price $K$ and maturity $T$ by $C_t(t,T,S_t,\sigma,K,r)$ and $P_t(t,T,S_t,\sigma,K,r)$ respectively. By definition, the expected gross returns for holding the options to expiration are given by:

$$R_{call} = \frac{E_t[\max(S_T - K, 0)]}{C_t(t,T,S_t,\sigma,K,r)} \tag{2.3}$$

$$R_{put} = \frac{E_t[\max(K - S_T, 0)]}{P_t(t,T,S_t,\sigma,K,r)}. \tag{2.4}$$

Propositions 1 and 2 indicate how these expected call and put option returns change with respect to the underlying volatility $\sigma$. We provide the detailed proof for the case of the call option in

Proposition 1, because the proof provides valuable intuition for the result. The intuition for the case of the put option is similar and the proof is relegated to the appendix.

**Proposition 1** *Everything else equal, the expected return of holding a call option to expiration is higher if the underlying asset has lower volatility* ($\frac{\partial R_{call}}{\partial \sigma} < 0$).

**Proof.** We start by reviewing several well-known facts that are needed to derive the main result. If the underlying asset follows a geometric Brownian motion, the price of a European call option with maturity $\tau = T - t$ written on the asset is given by the Black-Scholes-Merton formula:

$$\begin{aligned} C_t(t, T, S_t, \sigma, K, r) &= S_t N(d_1) - e^{-r\tau} K N(d_2) && (2.5) \\ d_1 &= \frac{\ln \frac{S_t}{K} + (r + \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}} \\ d_2 &= \frac{\ln \frac{S_t}{K} + (r - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}. && (2.6) \end{aligned}$$

Vega is the first-order derivative of the option price with respect to the underlying volatility. It measures the sensitivity of the option price to small changes in the underlying volatility. The Black-Scholes-Merton Vega is the same for call and put options:

$$\nu = \sqrt{\tau} S_t \psi(d_1) \tag{2.7}$$

where $\psi$ is the probability density function of the standard normal distribution. We also have:

$$S_t \psi(d_1) = e^{-r\tau} K \psi(d_2). \tag{2.8}$$

We first write the expected call option return in (2.3) in a convenient way. This allows us to conveniently evaluate the derivative of the expected option return with respect to the underlying volatility, using the Black-Scholes-Merton Vega in (2.7).

The denominator of (2.3) is the price of the call option and is therefore given by the Black-Scholes-Merton formula in (2.5). The numerator of (2.3), the expected option payoff at expiration, can be transformed into an expression that has the same functional form as the Black-Scholes-Merton formula. We get:

$$\begin{aligned} E_t[\max(S_T - K, 0)] &= \int_{z^*} (S_t e^{\mu\tau - \frac{1}{2}\sigma^2\tau + \sigma\sqrt{\tau}z} - K) \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz && (2.9) \\ &= e^{\mu\tau}[S_t N(d_1^*) - e^{-\mu\tau} K N(d_2^*)] && (2.10) \end{aligned}$$

where

$$z^* = \frac{\ln\frac{K}{S_t} - (\mu - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}} \qquad d_1^* = \frac{\ln\frac{S_t}{K} + (\mu + \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}} \qquad d_2^* = \frac{\ln\frac{S_t}{K} + (\mu - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}. \tag{2.11}$$

Combining (2.5) and (2.10), the expected return for holding a European call option to maturity is given by:

$$R_{call} = \frac{E_t[\max(S_T - K, 0)]}{C_t(t, T, S_t, \sigma, K, r)} = \frac{e^{\mu\tau}[S_t N(d_1^*) - e^{-\mu\tau} K N(d_2^*)]}{S_t N(d_1) - e^{-r\tau} K N(d_2)}. \tag{2.12}$$

Taking the derivative of (2.12) with respect to $\sigma$ gives:

$$\begin{aligned} \frac{\partial R_{call}}{\partial \sigma} &= \frac{e^{\mu\tau}\sqrt{\tau}S_t\psi(d_1^*)[S_t N(d_1) - e^{-r\tau}KN(d_2)] - e^{\mu\tau}[S_t N(d_1^*) - e^{-\mu\tau}KN(d_2^*)]\sqrt{\tau}S_t\psi(d_1)}{[S_t N(d_1) - e^{-r\tau}KN(d_2)]^2} \\ &= \frac{e^{\mu\tau}\sqrt{\tau}S_t\{\psi(d_1^*)[S_t N(d_1) - e^{-r\tau}KN(d_2)] - \psi(d_1)[S_t N(d_1^*) - e^{-\mu\tau}KN(d_2^*)]\}}{[S_t N(d_1) - e^{-r\tau}KN(d_2)]^2}. \end{aligned} \tag{2.13}$$

Note that we use equation (2.7) to derive (2.13). From (2.13) it can be seen that $\frac{\partial R_{call}}{\partial \sigma}$ inherits the sign of $EX = \psi(d_1^*)[S_t N(d_1) - e^{-r\tau}KN(d_2)] - \psi(d_1)[S_t N(d_1^*) - e^{-\mu\tau}KN(d_2^*)]$. We now show that $EX$ is negative. We have:

$$\frac{1}{\psi(d_1^*)\psi(d_1)}EX = \frac{S_t N(d_1) - e^{-r\tau}KN(d_2)}{\psi(d_1)} - \frac{S_t N(d_1^*) - e^{-\mu\tau}KN(d_2^*)}{\psi(d_1^*)}. \tag{2.14}$$

Using equation (2.8), it follows that

$$\frac{1}{\psi(d_1^*)\psi(d_1)}EX = \frac{S_t N(d_1) - \frac{S_t\psi(d_1)}{\psi(d_2)}N(d_2)}{\psi(d_1)} - \frac{S_t N(d_1^*) - \frac{S_t\psi(d_1^*)}{\psi(d_2^*)}N(d_2^*)}{\psi(d_1^*)} \tag{2.15}$$

$$= S_t[(\frac{N(d_1)}{\psi(d_1)} - \frac{N(d_2)}{\psi(d_2)}) - (\frac{N(d_1^*)}{\psi(d_1^*)} - \frac{N(d_2^*)}{\psi(d_2^*)})]. \tag{2.16}$$

According to economic theory, the expected rate of return on risky assets must exceed the risk-free rate ($\mu > r$). We therefore have $d_1^* > d_1$ and $d_2^* > d_2$. We also have $d_1^* - d_2^* = d_1 - d_2$ as well as $d_1^* > d_2^*$ and $d_1 > d_2$, from the definition of (2.6) and (2.11). Now consider $\frac{N(d)}{\psi(d)}$. It can be shown that it is an increasing and convex function of $d$. Evaluating $\frac{N(d)}{\psi(d)}$ at $d_1$, $d_2$, $d_1^*$, and $d_2^*$, it can be seen that the expression $(\frac{N(d_1)}{\psi(d_1)} - \frac{N(d_2)}{\psi(d_2)}) - (\frac{N(d_1^*)}{\psi(d_1^*)} - \frac{N(d_2^*)}{\psi(d_2^*)})$ effectively amounts to the negative of the second difference (derivative) of an increasing and convex function. Therefore:

$$(\frac{N(d_1)}{\psi(d_1)} - \frac{N(d_2)}{\psi(d_2)}) - (\frac{N(d_1^*)}{\psi(d_1^*)} - \frac{N(d_2^*)}{\psi(d_2^*)}) < 0. \tag{2.17}$$

This implies $EX < 0$ which in turn implies $\frac{\partial R_{call}}{\partial \sigma} < 0$. ■

**Proposition 2** *Everything else equal, the expected return of holding a put option to expiration is higher if the underlying asset has higher volatility ($\frac{\partial R_{put}}{\partial \sigma} > 0$).*

**Proof.** See Appendix B. ■

There is a subtle but important difference compared to the proof for instantaneous returns. In the instantaneous case, one exploits the fact that $\frac{N(x)}{\psi(x)}$ is an increasing function in $x$. In contrast, the finite-period derivation relies on the fact that $\frac{N(x)}{\psi(x)}$ is not only an increasing but also a convex function in $x$. This is required because any finite holding period option return is a nonlinear function of $\mu$, whereas the instantaneous return is a linear function of $\mu$.

These results can be extended to compute expected option returns over any holding period $h$ in the Black-Scholes-Merton model. Following Rubinstein (1984), the expected return on a call option is given by

$$\begin{aligned} R^h_{call} &= \frac{e^{\mu h}[S_0 N(d_1^*) - e^{-[r+(\mu-r)HP]T} K N(d_2^*)]}{S_0 N(d_1) - e^{-rT} K N(d_2)} \\ d_1^* &= \frac{\ln \frac{S_0}{K} + [HP(\mu - r) + r + \frac{1}{2}\sigma^2]T}{\sigma\sqrt{T}} \\ d_2^* &= \frac{\ln \frac{S_0}{K} + [HP(\mu - r) + r - \frac{1}{2}\sigma^2]T}{\sigma\sqrt{T}} \end{aligned} \tag{2.18}$$

where the timeline is shifted to $[0, T]$ from $[t, T]$ to ease notation, $h$ is the holding period ($0 < h < T$), and $HP = h/T$ is the ratio of the holding period to the life of the option contract. Details are provided in Appendix C. Note that the expected holding-to-expiration option return in (2.12) is nested in (2.18), for $HP = 1$. We can use the structure of the proof of Proposition 1 to show $\frac{\partial R^h_{call}}{\partial \sigma} < 0$, by observing $r + (\mu - r)HP > r$. Thus, we conclude that expected call (put) option returns decrease (increase) with underlying volatility for any holding period in the Black-Scholes-Merton model.

Figure 2 graphically illustrates these results for a realistic calibration of the Black-Scholes-Merton model. We set $\mu = 10\%$ and $r = 3\%$. We present results for out-of-the-money, at-the-money, and in-the-money options. The left-side panels are for calls and the right-side panels are for puts. Figure 2 clearly illustrates the qualitative results in Propositions 1 and 2. We discuss the quantitative implications in more detail in Section 6.2.

The patterns in expected returns suggest a simple interpretation of our results in terms of leverage, consistent with the intuition in Coval and Shumway (2001). As mentioned before, in equation (2.2), $\frac{\partial O_t}{\partial S_t}\frac{S_t}{O_t}$ reflects the leverage embedded in the option, which is a function of moneyness, maturity, and volatility. Coval and Shumway (2001) analyze index call and put returns as a function of the leverage due to moneyness. Figure 1 plots expected returns as a function of volatility for an ATM option with one month maturity. The expected return on the stock is $\mu = 10\%$. Expected call returns are positive and expected put returns are negative, following the arguments in Coval and Shumway (2001). For both calls and puts, the absolute value of returns is higher for the low-volatility options. This reflects leverage: low volatility options are cheaper and therefore constitute a more leveraged position.[2] Note that in the limit, as volatility goes to infinity, the expected call return approaches the stock return and the expected put return approaches the riskfree rate.

# 3 Data

We conduct two empirical exercises, one using the cross-section of equity option returns, and another one using the time series of index option returns. Here we discuss the two datasets used in these exercises. The sample period is from January 1996 to July 2013 for both datasets.

## 3.1 Equity Option Data

The main objective of our empirical exercise is to test Propositions 1 and 2 using the cross-section of options written on individual stocks. Propositions 1 and 2 predict a relation between expected option returns and underlying volatility, everything else equal. When studying the relation between option returns and the underlying volatility, it is therefore critical to control for other option characteristics that affect returns. Existing studies have documented that moneyness and maturity also affect option returns, see for example Coval and Shumway (2001).

To address this issue, we use option samples that are homogeneous in maturity and moneyness. For our benchmark empirical analysis, we use the cross-section of stock options that are at-the-money and one month away from expiration, because these are the most frequently traded options, and they are subject to fewer data problems (see, among others, Goyal and Saretto, 2009). In subsequent robustness exercises, we use options with different maturities and moneyness.

We obtain stock return data from CRSP and relevant accounting information from Compustat. We obtain option data from OptionMetrics through WRDS. OptionMetrics provides

[2] Higher volatility options are sometimes incorrectly thought of as more leveraged, presumably capturing the relation between unanticipated changes in volatility and higher prices. This argument ignores the cost of the option position.

historical option closing bid and ask quotes, as well as information on the underlying securities for U.S. listed index options and equity options. Every month, on the first trading day after monthly option expiration, we select equity options with $0.95 \leq K/S \leq 1.05$ that expire over the next month.[3] The expiration day for standard exchange-traded options is the Saturday immediately following the third Friday of the expiration month, so our sample consists mainly of Mondays. If Monday is an exchange holiday, we use Tuesday data.

We apply several standard filters to the option data. An option is included in the sample if it meets all of the following requirements: 1) The best bid price is positive and the best bid price is smaller than the best offer price; 2) The price does not violate no-arbitrage bounds. For call options we require that the price of the underlying exceeds the best offer, which is in turn higher than $max(0, S - K)$. For put options we require that the exercise price exceeds the best bid, which is in turn higher than $max(0, K - S)$; 3) No dividend is paid over the duration of the option contract; 4) Open interest is positive; 5) Volume is positive; 6) The bid-ask spread is higher than the minimum tick size, which is equal to \$0.05 when the option price is below \$3, and \$0.10 when the option price is higher than \$3; 7) The expiration day is standard, the Saturday following the third Friday of the month; 8) Settlement is standard; 9) Implied volatility is not missing.

We compute the monthly return from holding the option to expiration using the mid-point of the bid and ask quotes as a proxy for the market price of the option contract. If an option expires in the money, the return to holding the option to maturity is the difference between the terminal payoff and the initial option price divided by the option price. If an option expires out of the money, the option return is $-100\%$. Our equity option sample contains 247,859 call options and 188,046 put options over the time period from January 1996 to July 2013.[4]

In our benchmark results, we measure volatility using realized volatility computed using daily data for the preceding month, and we refer to this as 30-day realized volatility.[5] In the robustness analysis we use realized volatility over different horizons, which is also computed using daily data over the relevant horizon.

Table 1 reports summary statistics for equity options across moneyness categories. Moneyness

[3] We obtain similar results when we use options collected on the first trading day of each month.

[4] Stock options are American. We do not fully address the complex issue of early exercise, but attempt to reduce its impact by only including options that do not have an ex-dividend date during the life of the option contract. This of course does not address early exercise of put options (Barraclough and Whaley, 2012). However, several studies (see among others Broadie, Chernov, and Johannes, 2007; Boyer and Vorkink, 2014) argue that adjusting for early exercise has minimal empirical implications. See also the discussion in Goyal and Saretto (2009).

[5] Because this measure uses data for the previous month, it is effectively based on approximately 22 returns. For convenience, we refer to it as 30-day volatility. The same remark applies to volatility measures for other horizons used throughout the paper.

is defined as the strike price over the underlying stock price. On average the returns to buying call (put) options are positive (negative). Put option returns increase with the strike price. Call returns increase for the first four quintiles but decrease for the fifth.[6] Also note that option-implied volatility exceeds realized volatility for all moneyness categories, but the differences are often small. Gamma and Vega are highest for at-the-money options and decrease as options move away from the money.

### 3.2 Index Option Data

We also investigate the relation between volatility and expected returns using the time series of index option returns. On the first trading day after each month's option expiration date, we collect index options with moneyness $0.9 \leq K/S \leq 1.1$ that mature in the next month. Table 2 provides summary statistics for SPX option data by moneyness. Index put options (especially out-of-the-money puts) generate large negative returns, consistent with the existing literature (see for example Bondarenko, 2003). For example, for the moneyness interval $0.94 < K/S \leq 0.98$, the average return is $-40.6\%$ per month. Table 2 also shows that in our sample, out-of-the-money SPX calls have large negative returns. This is consistent with the results in Bakshi, Madan, and Panayotov (2010).

Comparing Tables 2 and 1 highlights several important differences between index options and individual stock options. First, the volatility skew, the slope of implied volatility against moneyness, is much less pronounced for individual stock options. Second, the average realized volatility for index options is approximately 17%, and therefore the volatility risk premium for index options exceeds the volatility risk premium for stock options. This is consistent with existing findings, but note that the index variance risk premium in our paper is smaller than many existing findings due to our sample period.

## 4 Volatility and the Cross-Section of Option Returns: Empirical Results

In this section, we empirically test Propositions 1 and 2 using the cross-section of options written on individual stocks. First, we present our benchmark cross-sectional results. As mentioned before, for our benchmark empirical analysis, we use the cross-section of stock options that are

[6] We verified that returns for further out-of-the money calls continue to decrease. Returns for calls are therefore non-monotonic as a function of moneyness, consistent with the results for index returns in Bakshi, Madan, and Panayotov (2010).

at-the-money and one month away from expiration to control for option characteristics other than volatility that affect returns. Subsequently, we conduct a series of tests to control for the expected returns on the underlying stocks. We also discuss the relation between volatility and straddle returns. Finally, the Black-Scholes-Merton model has some well-known empirical shortcomings, and it is possible that adjusting the theoretical model for these empirical shortcomings may affect the results. The most important shortcoming is the constant volatility assumption. We therefore investigate if our findings are robust to the presence of stochastic return volatility.

## 4.1 The Cross-Section of Option Portfolio Returns

Each month, on each portfolio formation date, we sort options with moneyness $0.95 \leq K/S \leq 1.05$ into five quintile portfolios based on their realized volatility, and we compute equal-weighted returns for these option portfolios over the following month. We conduct this exercise for call and put options separately.

Panel A of Table 3 displays the averages of the resulting time series of returns for the five call option portfolios, as well as the return spread between the two extreme portfolios. Portfolio "Low" contains call options with the lowest realized volatility, and portfolio "High" contains call options with the highest realized volatility. Proposition 1 states that the expected call option return is a decreasing function of the underlying stock volatility. Consistent with this result, we find that call option portfolio returns decrease monotonically with the underlying stock volatility. The average returns for portfolio High and portfolio Low are 0.9% and 14.7% per month respectively. The resulting return difference between the two extreme portfolios (H-L) is −13.8% per month and highly statistically significant, with a Newey-West (1987) t-statistic of −3.42.[7]

Panel B of Table 3 presents the averages of the resulting time-series of returns for the five put option portfolios. Again, portfolio Low (High) contains put options with the lowest (highest) underlying stock volatilities. For put option portfolios, the average return increases from −14.6% per month for portfolio Low to −7.5% per month for portfolio High, with a positive and significant H-L return difference of 7.1%. This finding confirms Proposition 2, which states that expected put option returns are an increasing function of the underlying stock volatility.

Table 3 also provides results using only options with moneyness $0.975 \leq K/S \leq 1.025$. By using a tighter moneyness interval, we reduce the impact of moneyness on expected option returns. The results are very similar. The average option portfolio returns decrease (increase) with the underlying stock volatility for calls (puts). The H-L differences are −13.8% and 7.7%

[7] T-statistics computed using the i.i.d. bootstrap as in Bakshi, Madan, and Panayotov (2010) are very similar.

for call and put option portfolios respectively, and are statistically significant. This indicates that our empirical results are not due to differences in option moneyness.[8]

These results are obtained using option returns computed using the mid-point of the bid and ask quotes. To ensure that our results do not depend on this assumption, Panel C of Table 3 computes average option portfolio returns based on the ask price. As expected, average returns are somewhat smaller than in Panels A and B. However, we again find a strong negative (positive) relation between call (put) option portfolio returns and the underlying stock volatility. The H-L differences are both statistically significant and are of a similar order of magnitude as the ones reported in Panels A and B.

Figure 3 complements the average returns in Table 3 by plotting the cumulative returns on the long-short portfolios over time. Figure 3 indicates that the negative (positive) sign for the call (put) long-short returns is quite stable over time, although it of course does not obtain for every month in the sample.

## 4.2 Controlling for Expected Stock Returns

The empirical results in Table 3 document the relation between volatility and expected option returns. These results control for other well-known determinants of option returns such as moneyness and maturity. Now we attempt to control for other confounding factors by analyzing the role of the drift $\mu$ in the law of motion for the underlying asset (2.1).

We derive Proposition 1 and 2 assuming a constant drift $\mu$. If the drift is not constant, our empirical results in Table 3 may be due to patterns in the returns on the underlying stocks rather than to the mechanics of option returns studied in Propositions 1 and 2. We now discuss this in more detail. First consider a drift $\mu$ that depends on volatility. We know that $\frac{\partial R_{call}}{\partial \mu} > 0$ and $\frac{\partial R_{put}}{\partial \mu} < 0$ (see Appendix D for details). We therefore need to refer to the theoretical and empirical literature on the relation between volatility and expected stock returns. If the relation between stock returns and volatility is positive, it cannot explain the empirical relation documented in Table 3. If this relation is negative, on the other hand, we need to control for it in the empirical analysis.

Theory predicts a positive relation between stock returns and volatility, but the empirical time-series evidence is tenuous, perhaps because estimating expected returns from the time

[8] We focus on patterns in expected returns as a function of volatility. We do not address the more complex question of the riskiness of these returns. Not surprisingly, the standard deviation of returns is lower for higher volatility quintiles, but these differences are small compared to differences in returns. The pattern in Sharpe ratios is therefore similar to the pattern in returns. However, Sharpe ratios are a poor measure of risk for option strategies.

series of returns is notoriously difficult.[9] In the cross-sectional literature, Ang, Hodrick, Xing, and Zhang (2006) document a negative relation between volatility and stock returns. Their work has inspired a voluminous literature, and some studies find a positive or insignificant cross-sectional relation, but overall the literature confirms their findings.[10] We therefore need to re-visit our results while controlling for the expected return on the underlying security. It is of course well-known that controlling for the expected return on the underlying stock is difficult. To the extent that we are not able to do so, it is possible that the cross-sectional effect documented by Ang, Hodrick, Xing, and Zhang (2006) partly explains our results.

We now present empirical results that control for the expected return on the underlying security in various ways. First we present results for double sorts on volatility and average historical stock return. Second, we specify a single-factor market model for the underlying security and control for the underlying stock's exposure to the market. Third, we use Fama-MacBeth regressions to control for a wide variety of determinants of expected stock returns. Fourth, we use the option pricing model to control for the empirical differences in stock returns between quintiles.

### 4.2.1 Controlling for Expected Stock Returns Using Historical Averages

Expected call (put) option returns increase (decrease) with the expected return on the underlying asset. If the high volatility portfolios in Table 3 are primarily composed of stocks that have lower expected returns than those in the low volatility portfolios, the result that average call (put) options in the high volatility portfolios earn lower (higher) returns may not be due to volatility. We therefore start by documenting if the underlying stock returns affect our results by empirically controlling for expected stock returns. This is of course challenging because unlike volatility, expected stock returns are notoriously difficult to measure.

Our first approach follows Boyer and Vorkink (2014), who estimate expected stock returns as the simple average of daily returns over the past six months. Each month we first form five quintile portfolios based on estimated expected stock returns $\mu$, and then within each $\mu$ quintile options are further sorted into five quintile portfolios according to underlying stock volatility. We once again measure underlying stock volatility by 30-day realized volatility.

Table 4 presents the results of this double sort. The columns correspond to different volatility

[9] See, among others, Nelson (1991), Campbell and Hentschel (1992), French, Schwert, and Stambaugh (1987), Glosten, Jagannathan and Runkle (1993), Goyal and Santa-Clara (2003), Ghysels, Santa-Clara and Valkanov (2005), Bali et al. (2005), and Bali (2008).

[10] See, among many others, Adrian and Rosenberg (2008), Ang, Hodrick, Xing, and Zhang (2009), Bali and Cakici (2008), Chen and Petkova (2012), Fu (2009), Huang, Liu, Rhee, and Zhang (2009), and Stambaugh, Yu, and Yuan (2015).

levels, and the rows correspond to different average returns. Consistent with the single sort results, in each $\mu$ quintile call (put) option portfolio returns decrease (increase) with underlying volatility. In all $\mu$ quintiles, the average return differences between the two extreme call option portfolios are negative, ranging from $-24\%$ to $-11\%$ per month, and highly significant. For put options, the high minus low differences are all positive and statistically significant in four out of five $\mu$ quintiles. These findings suggest that our results are not driven by differences between the expected returns of the underlying stocks.

#### 4.2.2 A Single-Factor Market Model

Estimates of expected returns from historical averages as in Section 4.2.1 are notoriously imprecise. Our next approach controls for expected returns using the simple market or index model rather than the historical average. Panel A of Table 5 presents results for a double sort on market beta and volatility. Beta is estimated by the market model over the most recent 30 days preceding the portfolio formation date. The results are similar to those in Table 4, where we control for the expected return using lagged average returns, but the t-statistics are somewhat smaller. Average call option returns decrease with volatility for each beta quintile and the return spread between the two extreme portfolios is statistically significant across all beta quintiles. In contrast, average put option returns increase with volatility for each beta quintile and the return spread is significant for the top three beta quintiles.

Panel B of Table 5 uses the results from the market model in a slightly different way. We present results for sorts on idiosyncratic volatility based on the market model. Panel B indicates a negative relation between call option portfolio returns and idiosyncratic volatility, and a positive relation between put option portfolio returns and idiosyncratic volatility.[11] We obtain similar results when sorting on idiosyncratic volatility computed relative to the Fama-French three-factor model.

#### 4.2.3 Fama-MacBeth Regressions

To control as comprehensively as possible for the impact of the drift of the underlying assets on option returns, we run Fama-MacBeth (1973) regressions that allow us to simultaneously control for risk factors and stock characteristics that have been shown in the existing literature to be related to expected stock returns.

[11] Given the additional assumption of the market model, the results in Propositions 1 and 2 effectively establish a relation between option returns and idiosyncratic volatility. This interpretation is more in line with Johnson's (2004) analysis of the role of volatility in returns on levered equity.

Every month we run the following cross-sectional regression

$$R^i_{t+1} = \gamma_{0,t} + \gamma_{1,t} VOL^i_t + \Phi_t Z^i_t + \epsilon \tag{4.1}$$

where $R^i_{t+1}$ is the return on holding option $i$ from month $t$ to month $t+1$, $VOL^i_t$ is the underlying stock volatility for option $i$, and $Z^i_t$ is a vector of control variables that includes the stock's beta, firm size, book-to-market, momentum, stock return reversal, the option skew, the volatility risk premium, the slope of the implied volatility term structure, as well as option characteristics such as moneyness, Delta, Vega, Gamma and option-beta. Option beta is defined as delta times the stock price divided by the option price. Both $VOL^i_t$ and $Z^i_t$ are observable at time $t$ for option $i$. We again use 30-day realized volatility as a proxy for the underlying stock volatility.

Table 6 reports the time-series averages of the cross-sectional $\gamma$ and $\Phi$ estimates from equation (4.1), along with Newey-West (1987) t-statistics which adjust for autocorrelation and heteroscedasticity. Columns (1) to (3) report regression results for call options. Column (1) of Table 6 shows that in a univariate regression the average slope coefficient on 30-day realized volatility is $-0.239$ with a Newey-West t-statistic of $-4.19$. This estimate is consistent with the sorting results. The difference in the average underlying volatility between the two extreme call option portfolios in Table 3 is 0.6, which implies a decline of $-0.239 \times 0.6 = 14.34\%$ per month in average returns if a call option were to move from the bottom volatility portfolio to the top volatility portfolio, other characteristics held constant. This estimate is very similar to the result in Table 3.

The specification in column (2) includes several well-known determinants of cross-sectional stock returns. The loading on volatility increases in absolute value from -0.239 to -0.277 and remains highly significant. The specification in column (3) includes additional controls as well as option characteristics. The slope coefficient on volatility is even larger in absolute value and is again statistically significant. The results in columns (1)-(3) are all consistent with our theoretical conjecture in Proposition 1.

In columns (4)-(6), we provide results for put options. As expected, the average slope coefficient on underlying volatility is positive and statistically significant for all specifications, ranging from 11.7% to 58.4% per month. These findings again suggest that our results cannot be attributed to differences in expected stock returns. As we add more controls in column (6), the loading on volatility increases significantly in absolute value.

We conclude that the results in Table 6 are consistent with the theoretical predictions. Moreover, the empirical results get stronger when we insert more controls for expected stock returns. For call options, the slope coefficient on 30-day realized volatility is $-0.389$ in column (3), compared to $-0.239$ in column (1). For put options, the estimate in column (6) is 0.584, compared to 0.117 in column (4). This may suggest that we control more effectively for the effect of the drift of the underlying security when we include more controls. Presumably controlling for the drift using expected returns or a market model as in Sections 4.2.1 and 4.2.2 is not very effective, which explains why the results in Tables 4 and 5 are very similar to the benchmark results in Table 3. However, note that the t-statistic in column (3) is lower than that in column (1), and therefore some caution is advisable when interpreting these results.

In columns (3) and (6), we also include the variance risk premium studied by Goyal and Saretto (2009) and the slope of the volatility term structure studied by Vasquez (2012). The variance risk premium is significant for puts and the slope is significant for both calls and puts. However, note that Goyal and Saretto (2009) and Vasquez (2012) study delta-hedged returns, and we analyze raw returns. From our perspective, the most important conclusion is that the cross-sectional relation between volatility and returns remains in the presence of these controls.

#### 4.2.4 A Controlled Experiment

Finally, we assess if Ang, Hodrick, Xing, and Zhang's (2006) finding of a negative relation between volatility and stock returns can explain our results in a more direct way using a simple computation. In Table 3, the difference in option returns between the fifth and the first quintiles is -13.8% for call options (0.9%-14.7%). We compute the average returns on the stocks in these portfolios, which is 10.8% for the first quintile and 4.8% in the fifth quintile. We now fix volatility $\sigma$ across these quintiles to conduct a controlled experiment and compute option returns using the Black Scholes-Merton model. Consider a fixed $\sigma$ of 50%, which is close to our sample average. This experiment indicates how much of the return differential in option returns is generated by the differential in returns in the underlying stocks.

For the low volatility quintile, the average return of 10.8% and the 50% volatility yield a monthly option return of 6.31%. For the high volatility quintile the average return is 4.8%, which for a 50% volatility gives a 1.63% option return. The difference between the two returns is $1.63\% - 6.31\% = -4.68\%$. In other words, this computation indicates that of the 13.8% return differential in the data, 4.68% is due to the differences in stock returns. The volatility difference accounts for the majority of the difference in option returns, empirically confirming the theoretical relation.

## 4.3 Volatility and Expected Straddle Returns

Straddle returns are not very sensitive to the expected returns on the underlying security. Therefore, several existing papers that investigate the cross-sectional relation between option returns and different aspects of volatility focus on straddle returns to separate the cross-sectional effect of volatility and volatility-related variables from that of the underlying stock returns. See for example Goyal and Saretto (2009) and Vasquez (2012).

A straddle consists of the simultaneous purchase of a call option and a put option on the same underlying asset. The call and put options have the same strike price and time to maturity. The expected gross return on a straddle is given by:

$$R_{straddle} = \frac{E_t[\max(S_T - K, 0)] + E_t[\max(K - S_T, 0)]}{C_t(\tau, S_t, \sigma, K, r) + P_t(\tau, S_t, \sigma, K, r)}$$

where $C_t(\tau, S_t, \sigma, K, r)$ and $P_t(\tau, S_t, \sigma, K, r)$ are the call and put prices that an investor has to pay to build a long position in straddle.

Because the derivative of calls and puts with respect to volatility has the opposite sign, it is impossible to obtain general results for straddles. Appendix E shows that $d_2 > 0$ is a sufficient condition for a negative relation between straddle returns and underlying volatility. Recall that $d_2 = \frac{\ln \frac{S_t}{K} + (r - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}$. The condition $d_2 > 0$ is thus likely to hold for straddles with strike prices below the current stock price, and we investigate if average straddle returns decrease with underlying volatility for such straddles. Table A.1 confirms that this relation indeed holds in the data. However, effectively the return on the straddle is dominated by the call option when $d_2 > 0$, which means that the negative sign in theory and in the data simply confirms the results above.

## 4.4 Stochastic Volatility and Expected Option Returns

The Black-Scholes-Merton model's treatment of volatility is perhaps its most important shortcoming. An extensive literature has demonstrated that volatility is time varying, and that (the innovations to) volatility and stock returns are correlated.[12] This correlation is often referred to as the leverage effect.

To address the implications of time-varying volatility and the leverage effect, we now analyze expected option returns using a stochastic volatility model instead of the Black-Scholes-Merton model. We use the Heston (1993) model, which has become the benchmark in this literature

[12] For seminal contributions to this literature, see Engle (1982), Bollerslev (1986), Nelson (1991), Glosten, Jagannathan and Runkle (1993), and Engle and Ng (1993).

because it captures important stylized facts such as time-varying volatility and the leverage effect, while also allowing for quasi-closed form European option prices. The Heston (1993) model assumes that the asset price and its spot variance obey the following dynamics under the physical measure $P$:

$$
\begin{aligned}
dS_t &= \mu S_t dt + S_t \sqrt{V_t} dZ_1^P \\
dV_t &= \kappa(\theta - V_t)dt + \sigma \sqrt{V_t} dZ_2^P
\end{aligned}
$$

where $\mu$ is the drift of the stock price, $\theta$ is the long run mean of the stock variance, $\kappa$ is the rate of mean reversion, $\sigma$ is the volatility of volatility, and $Z_1$ and $Z_2$ are two correlated Brownian motions with $E[dZ_1 dZ_2] = \rho dt$.

By focusing on discrete holding periods instead of instantaneous returns, we can express expected returns in the Heston model in quasi-closed form.[13] Appendix F shows that the expected return of holding a call option to expiration in the Heston model is given by:

$$
R_{call}^{Heston}(S_t, V_t, \tau) = \frac{e^{\mu\tau}[S_t P_1^* - e^{-\mu\tau} K P_2^*]}{S_t P_1 - e^{-r\tau} K P_2} \tag{4.2}
$$

where $P_1$, $P_2$, $P_1^*$ and $P_2^*$ are defined in Appendix F. The expected call option return in the Heston model has the same functional form as in the Black-Scholes-Merton model, but unlike for the Black-Scholes-Merton model, the sign of $\frac{\partial R_{call}^{Heston}(S_t, V_t, \tau)}{\partial V_t}$ cannot be derived analytically. However, the expected option return in equation (4.2) can be easily calculated numerically given a set of parameter values.

In Panels B and C of Table 7, we compute expected option returns according to (4.2) for different parameterizations of the expected stock return $\mu$ and the conditional stock variance. For simplicity we first set the variance risk premium $\lambda$ equal to zero. For all other parameters, we use the parameters from Broadie, Chernov, and Johannes (2009), which are listed in Panel A. The patterns in expected option returns in a stochastic volatility model are similar to the patterns in Black-Scholes-Merton expected option returns. In particular, expected call option returns increase (decrease) with expected stock return (current stock variance), whereas expected put option returns decrease (increase) with expected stock return (current stock variance). In unreported results, we obtain similar results using different parameterizations.

In the Black-Scholes-Merton model, volatility affects expected returns through leverage. In the Heston model, volatility affects expected returns not only through leverage, but also through the volatility risk premium $\lambda$. Figure 4 further explores expected returns in the Heston model.

[13] The delta and vega are not available in closed form when computing instantaneous option returns.

We set $\lambda$ equal to 0 or $-0.5$. The main conclusion is that expected returns do not strongly depend on $\lambda$. The relation between volatility and expected option returns is similar to the results in the Black-Scholes-Merton model.[14]

Finally, Panel D of Table 7 shows straddle returns as a function of the volatility risk premium $\lambda$. Returns increase with higher volatility risk premiums. This is consistent with the empirical findings in Goyal and Saretto (2009), who document that option returns increase as a function of the variance risk premium.

# 5 Robustness

In this section we investigate the robustness of the results in Table 3 to a number of implementation choices. We investigate the robustness of the empirical results to the measurement of realized volatility, the composition of the option sample, and the weights used to compute portfolio returns. We also use holding-period returns rather than holding-to-maturity returns.

## 5.1 The Volatility Measure

Table 3 uses realized volatility computed using daily data for the preceding month as a measure of the underlying volatility. This is a standard volatility measure that is often used in the literature. Ang, Hodrick, Xing, and Zhang (2006) and Lewellen and Nagel (2006) argue that 30-day realized volatility strikes a good balance between estimating parameters with a reasonable level of precision and capturing the conditional aspect of volatility. We now consider five alternative estimators of underlying stock volatility. We proxy underlying volatility using realized volatilities computed over the past 14 days, the past 60 days, and the past 365 days, as well as option-implied volatility and a simple autoregressive AR(1) model for volatility to take into account the mean reversion in volatility.

Panel A of Table 8 presents average returns for the five quintile call option portfolios and Panel B reports average returns for put option portfolios. Consistent with our benchmark results in Table 3, we find that for all underlying volatility proxies, the returns on the call option portfolios exhibit a strong negative relation with underlying stock volatilities, while put option portfolio returns display a strong positive relation with underlying stock volatilities. For example, when sorting on 60-day realized volatility, the average returns for call option portfolios with the largest and smallest underlying volatilities are 1.4% and 15.5% per month respectively. The resulting difference between the two extreme portfolios is $-14.1\%$ per month and is highly statistically

[14]For (unrealistically) large negative $\lambda$, the relation is not monotone for OTM options.

significant with a Newey-West t-statistic of $-3.44$. For put option portfolios, the average returns monotonically increase from $-15.7\%$ per month for the lowest volatility portfolio to $-5.9\%$ per month for the highest volatility portfolio. The resulting difference is 9.8% per month and is also statistically significant.

When sorting on 14-day and 365-day realized volatility, the returns display a similar pattern. The average returns decrease (increase) with underlying volatilities for call (put) portfolios. The return differences between the two extreme call option portfolios are negative and statistically significant with a magnitude of $-13\%$ and $-8.6\%$ per month, respectively. The corresponding differences for put option portfolios are positive and statistically significant, with a magnitude of 5.9% and 11.7% per month, respectively.

We also sort options based on implied volatilities. Option-implied volatilities are attractive because they provide genuinely forward-looking estimates, but they are model-dependent and may include volatility risk premiums.[15] Again consistent with our benchmark results, we find that call (put) option portfolios with larger implied volatilities earn lower (higher) returns. Panel A of Table 8 reveals that returns on call option portfolios monotonically decrease with implied volatilities. The return spread is $-16.6\%$ per month and is highly statistically significant. The return spread for the two extreme put option portfolios is positive with a magnitude of 5.9% per month, but it is not statistically significant. Finally, we use an autoregressive model for volatility instead. In particular, we obtain an estimate of conditional volatility by fitting an AR (1) model on monthly realized volatilities. The results are again statistically significant. The economic magnitude is somewhat smaller for calls and somewhat larger for puts.

These results suggest that our empirical findings are not due to the volatility measure used in Table 3.

## 5.2 The Option Sample

We now investigate the relation between expected option returns and underlying volatility using five other option samples with different maturity and moneyness. We examine the following five option samples: two-month at-the-money options, one-month in-the-money options, two-month in-the-money options, one-month out-of-the-money options, and two-month out-of-the-money options. We define at-the-money as having moneyness of $0.95 \leq K/S \leq 1.05$, in-the-money calls as $0.80 \leq K/S < 0.95$, and in-the-money puts as $1.05 < K/S \leq 1.20$. Out-of-the-money calls are defined as $1.05 < K/S \leq 1.20$ and out-of-the-money puts as $0.80 \leq K/S < 0.95$.

[15] On the volatility risk premium embedded in individual stock options, see Bakshi and Kapadia (2003b), Driessen, Maenhout, and Vilkov (2009), and Carr and Wu (2009) for more details.

Table 9 presents the results. Panel A of Table 9 provides average returns of call option portfolios sorted on 30-day realized volatility for the five alternative option samples. Consistent with the benchmark results in Table 3, we find that returns on call option portfolios decrease with underlying volatility for all option samples. The return differences between the two extreme portfolios are negative and statistically significant in all cases, with magnitudes ranging from $-7.8\%$ to $-18.6\%$ per month. For instance, for two-month at-the-money calls, the equal-weighted average option portfolio returns decrease monotonically with underlying volatility. The return spread is $-17.1\%$ per month and highly significant with a Newey-West t-statistic of $-3.04$.

Panel B of Table 9 presents average returns of put option portfolios sorted on 30-day realized volatility for the five option samples. Average put option returns exhibit a strong positive relation with underlying volatilities. The returns spreads are all positive and statistically significant, ranging from 5.7% to 17.8% per month. For instance, for two-month at-the-money puts, average returns monotonically increase from $-20.7\%$ per month for the lowest volatility portfolio to $-5.6\%$ per month for the highest volatility portfolio. The resulting return spread is 15.1% per month and is both economically and statistically significant.

For empirical results that use index options, using out-of-the-money options is very important because this market is more liquid and has higher volume, as evidenced by Table 2. For equity options, the differences in liquidity and volume across moneyness are less pronounced, as evidenced by Table 1. Nevertheless, it is reassuring that the results are robust when we only use out-of-the-money options in Table 9.

These results suggest that our empirical findings are not due to the sample used in Table 3.

## 5.3 The Portfolio Weighting Method

In this subsection, we examine if the negative (positive) relation between call (put) option portfolio returns and underlying volatility persists if different weighting methods are used for computing option portfolio returns. We calculate option volume weighted, option open interest weighted and option value weighted average portfolio returns. Option value is defined as the product of the option's open interest and its price.[16]

Table A.2 contains return spreads for option portfolios sorted on 30-day realized volatility, using these alternative weighting methods. Regardless of the weighting method, the return spreads are negative (positive) for call (put) option portfolios, and they are statistically significant in most cases. These results suggest that our empirical findings are not due to the equal-weighting method used in Table 3.

[16] We also consider portfolio returns weighted by underlying stock capitalization and find similar results.

## 5.4 Holding-Period Returns

Ni, Pearson, and Poteshman (2005) argue that holding-to-maturity option returns are affected by biases at expiration. We therefore repeat the analysis in Table 3 using one-month option returns instead of holding-to-maturity returns.[17] Table A.3 presents the results for ATM, ITM, and OTM call and put options. The results are again statistically significant and consistent with Propositions 1 and 2. However, the magnitudes of the long-short returns are smaller, especially for calls.

# 6 Discussion and Extensions

In this section, we further explore our results. We first discuss delta-hedged returns, which have been studied in the existing literature, and we verify if the differences in expected returns between portfolios are consistent with theoretical predictions. Subsequently we investigate if the models' quantitative implications for returns are consistent with the data and we compute option-implied average stock returns. Finally we provide a detailed discussion of the differences between our results and those in the existing literature.

## 6.1 Delta-Hedged Returns

Cao and Han (2013) document that the cross-sectional relation between idiosyncratic stock volatility and both delta-hedged call and put option returns is negative.[18] It is natural to wonder if these empirical results are consistent with our theoretical and empirical results, especially because the results seem so different.

We show that these results are mutually consistent, and simply result from the difference between raw and delta-hedged returns. Our study may seem superficially related to Cao and Han (2013) but the analysis is fundamentally different. Our study empirically investigates the theoretical relation between option returns and the underlying stock volatility, which by definition is accounted for when computing delta-hedged returns. When we investigate the robustness of our results in Section 4.2, we correct for the drift of the underlying stock, whereas delta hedging by definition corrects for the entire underlying return, which includes the drift as well as the diffusive part.

[17] Broadie, Chernov, and Johannes (2009) argue against the use of holding period returns. Duarte and Jones (2007) argue that bid-ask bounce can bias returns, which also favors the use of holding to-maturity returns.

[18] See also Black and Scholes (1972) on the relation between delta-hedged option returns and volatility.

Panel A of Table 10 reports on delta-hedged returns for portfolios sorted on volatility. We repeat the analysis in Cao and Han (2013) with two differences in implementation. First, we use total volatility instead of idiosyncratic volatility because the focus of our study is on total volatility and we want to stay closer to the results in Table 3. Second, while Cao and Han (2013) rebalance daily, we use static hedging for reasons to be explained below. We verified that these differences in implementation do not significantly affect the results. Panel A demonstrates the robustness of the results in Cao and Han (2013). Consistent with their results, we find a statistically significant negative relation between volatility and both call and put returns.

Bollen and Whaley (2004) and Cao and Han (2013) emphasize market frictions and inventory management as plausible explanations for the negative relation between volatility and delta-hedged call and put returns. Bakshi and Kapadia (2003a, 2003b) show that delta-hedged option returns can be used to infer the market price of volatility risk if volatility is stochastic. Our implementation uses static hedging to emphasize an additional potential explanation for these results that differs from these existing explanations. In the Black-Scholes-Merton framework, delta-hedged option returns are exactly zero in the ideal case of continuous trading. In practice, however, it is impossible to rebalance the portfolio continuously. It is well understood (see for instance Branger and Schlag, 2008; Broadie, Chernov and Johannes, 2009) that empirical investigations using delta-hedged returns must be interpreted with caution, not only due to model misspecification, but also due to discretization errors and transaction costs.

Consider how underlying volatility impacts discretely delta-hedged option returns in the Black-Scholes-Merton model in the case of static hedging. We form the delta-hedged portfolio at time $t$ and keep it unadjusted until the expiration date of the option at $T$. The delta-hedged return for a call option, which can also be interpreted as the hedging error, is given by

$$\begin{aligned}\Pi^C_{t,T} &= C_T - C_t - \Delta_t(S_T - S_t) - (C_t - \Delta_t S_t)(e^{r\tau} - 1) &(6.1)\\ &= C_T - \Delta_t S_T - (C_t - \Delta_t S_t)e^{r\tau}. &(6.2)\end{aligned}$$

where $C_t$ and $C_T$ are call option prices at time $t$ and $T$, $S$ is the stock price, and $r$ again denotes the instantaneous risk-free rate. Using the results for the expected option payoff at maturity $E_t(C_T)$, we get

$$\begin{aligned}E_t(\Pi^C_{t,T}) &= E_t[C_T - \Delta_t S_T - (C_t - \Delta_t S_t)e^{r\tau}]\\ &= e^{\mu\tau}[S_t N(d_1^*) - e^{-\mu\tau} K N(d_2^*)] - \Delta_t S_t e^{\mu\tau} - (C_t - \Delta_t S_t)e^{r\tau}\\ &= e^{\mu\tau}[S_t N(d_1^*) - e^{-\mu\tau} K N(d_2^*)] - N(d_1) S_t e^{\mu\tau} + K N(d_2). &(6.3)\end{aligned}$$

The expected delta-hedged return (or hedging error) for a put option is given by

$$E_t(\Pi_{t,T}^P) = e^{\mu\tau}[e^{-\mu\tau}KN(-d_2^*) - S_tN(-d_1^*)] + N(-d_1)S_te^{\mu\tau} - KN(-d_2). \tag{6.4}$$

To understand how underlying volatility affects delta-hedged option returns, we need the sign of the partial derivatives of $E_t(\Pi_{t,T}^C)$ and $E_t(\Pi_{t,T}^P)$ with respect to $\sigma$. First, note that equation (6.4) is equivalent to equation (6.3) by put-call parity. We therefore focus on the relation between the expected delta-hedged call option return and underlying volatility. The partial derivative $\frac{\partial E_t(\Pi_{t,T}^C)}{\partial\sigma}$ is available analytically, but can either be positive or negative depending on the underlying parameters. However, we can easily evaluate expected returns numerically for a given set of parameters.

Table A.4 shows that this negative relationship continues to hold and is strengthened when including other control variables in a Fama-MacBeth regression. Table A.4 also shows indicates that the results of Goyal and Saretto (2009) are confirmed for our sample: delta-hedged returns on puts and calls are positively related to the variance risk premium.

Panel B of Table 10 considers static delta-hedging of a hypothetical call option with strike price of \$100 and a one-month maturity. We report expected delta-hedged returns for different values of $S$, $\mu$, and $\sigma$. Expected delta-hedged returns are all positive, which is consistent with the finding of Branger and Schlag (2008) that discretization error induces positive expected hedging error in the Black-Scholes-Merton model. More importantly for our purpose, the expected delta-hedged return decreases with underlying volatility for all parameter combinations.

We conclude that in the context of the Black-Scholes-Merton model, static delta-hedging will result in a negative relation between delta-hedged option returns and underlying volatility for plausible parameterizations of the model. This qualitative result also obtains in any practical situation where hedging is conducted in discrete time, for instance when the hedge is rebalanced daily. Most importantly for our conclusions, the negative relation between volatility and delta-hedged call and put returns is consistent with the negative (positive) relation between volatility and call (put) returns. Both cross-sectional relations are supported by the simple analytics of the Black-Scholes-Merton model.

## 6.2 Volatility and Expected Option Returns: A Quantitative Assessment

So far we have limited ourselves to empirically verifying the qualitative predictions in Propositions 1 and 2. We now go one step further and assess the magnitude of the return difference for portfolios with different underlying volatility.

The first row of Panels A and B of Table 11 reports the benchmark results from Table 3. The call option quintile portfolio with high volatility earns 0.9% per month and the call option quintile portfolio with low volatility earns 14.7% per month. The put option quintile portfolio with high volatility earns −7.5% per month and the put option quintile portfolio with low volatility earns −14.6% per month. We assess if these return differences are consistent with theory by computing expected returns and volatility for the underlying stocks for these portfolios, and computing expected returns using the Black-Scholes-Merton model.

Using historical averages for our sample period 1996-2013, we obtain an annualized $\mu$ of 4.8 percent and volatility $\sigma$ of 81.3 percent for the high volatility call option portfolio. For the low volatility call option portfolio, we obtain an annualized $\mu$ of 10.8 percent and volatility $\sigma$ of 20.9 percent. These results are reported in the second and third rows of Panel A. Assuming a 3% annual interest rate, the Black-Scholes-Merton model predicts a expected option return of 14.5% per month for the low volatility call option portfolio and 1.1% per month for the high volatility call option portfolio. These results are reported in the fourth rows of Panel A. These expected option returns are very close to the average returns in the first row.

The second and third rows of Panel B indicate that for the low volatility put option portfolio, the underlying annualized $\mu$ is 10.8% and the underlying volatility $\sigma$ is 21.4%. The high volatility put option portfolio has a $\mu$ of 4.8% and a $\sigma$ of 82.7%.[19] Again using the Black-Scholes-Merton model, the fourth row shows that this gives expected option returns of −12.4% and −0.5% per month. For the low volatility portfolio, the expected return is close to the sample average in the first row, but this is not the case for the high volatility put portfolio.

Overall, we conclude that the implied call option returns are close to what we observe in the data on average, despite the well-known shortcomings of the Black-Scholes-Merton model. The results for put options are less impressive than those for calls, which may be due to the well-known stylized fact that put options are expensive, possibly due to demand pressure (see Bollen and Whaley, 2004).

## 6.3 Option-Implied Returns and Volatility

The qualitative difference between the results for call and put options in Section 6.2 can equivalently be expressed in terms of option-implied returns. The last row of Panels A and B of Table 11 presents the results of this exercise. We use the average volatility in each quintile and then

[19] Note that the average stock returns and the average stock volatilities of the five quintile portfolios are slightly different for calls and puts. This is because for some stocks we have calls but not puts and vice versa.

invert the Black-Scholes-Merton formula to obtain an estimate of the implied $\mu$.

Table 11 indicates that the returns implied by call options are very close to the actual average stock returns, but this is not the case for the returns implied by puts. This result is essentially the mirror image of the finding discussed in Section 6.2.

### 6.4 Further Discussion and Related Literature

We do not provide an overview of the entire related literature on empirical option pricing, because it is vast and our results are easily distinguished.[20] However, our results are at the intersection of several strands of empirical research on cross-sectional asset pricing. We now discuss some of these related studies in more detail in order to highlight our specific contribution.

The literature characterizing volatility in index returns and stock returns is well-known and also too vast to cite here. Our paper is most closely related to a series of papers that highlight one particular dimension of this literature, namely the cross-sectional relation between volatility and expected stock returns. Even in this cross-sectional literature, it is important to differentiate between studies that investigate (aggregate) volatility as a pricing factor in the cross-section of returns and studies that investigate stock returns as a cross-sectional function of their own idiosyncratic or total volatility. Ang, Hodrick, Xing, and Zhang (2006) investigate both issues. As discussed in Section 4.2, our contribution is clearly more related to their investigation of the cross-sectional relation between the stock's own (lagged) volatility and returns.

There is also a growing literature on the cross-sectional relation between option-implied information and stock returns. Once again, some papers use index options to extract marketwide information on pricing factors for the cross-section of stock returns, while other papers use equity options to extract firm-specific information that can be used as a cross-sectional predictor of returns.[21]

Our paper differs from all of these studies because it investigates the relation between the volatility of the underlying (the stock) and the cross-section of option returns. The literature on the cross-section of equity option returns has also grown rapidly.[22] Boyer and Vorkink (2014)

[20] See Bates (2003) and Garcia, Ghysels, and Renault (2010) for excellent surveys on empirical option pricing.

[21] Chang, Christoffersen, and Jacobs (2013) use option-implied index skewness as a pricing factor. Conrad, Dittmar and Ghysels (2013) study the relation between stock returns and volatility, skewness and kurtosis extracted from equity options. The factor used by Ang, Hodrick, Xing, and Zhang (2006) is actually the VIX, so strictly speaking it is about option-implied information as a factor. For additional work, see, for example, Bali and Hovakimian (2009), Cremers and Weinbaum (2010) and Xing, Zhang, and Zhao (2010).

[22] Another literature focus on explaining the cross-section of option prices or implied volatilities rather than option returns. See for instance Duan and Wei (2008) and Bollen and Whaley (2004). An, Ang, Bali, and Cakici (2014) document that stock returns are higher (lower) following increases in call (put) implied volatility, but also link past stock returns with future option-implied volatility.

document a negative relation between ex-ante option total skewness and future option returns. Goodman, Neamtiu, and Zhang (2013) find that fundamental accounting information is related to future option returns. Karakaya (2014) proposes a three-factor model to explain the cross-section of equity option returns. Linn (2014) finds that index volatility is priced in the cross-section of option returns. Several recent papers use option valuation models to highlight cross-sectional differences between equity options.[23] As discussed in Section 6.1, our work is also related to a series of recent papers that document interesting patterns in the cross-section of delta-hedged option returns related to the volatility of the underlying securities (Goyal and Saretto, 2009; Vasquez, 2012).

We contribute to this growing literature on the cross-section of option returns by highlighting the theoretically expected relation between expected option returns and stock volatility. Stock volatility is often included in a cross-sectional study of option *returns*, because it is a well-known determinant of option *prices*. By being explicit about the relation between volatility and call and put returns, our work not only suggests that empirical work on option returns should control for the effect of volatility when identifying other determinants of option returns, it also predicts the sign of the relation between volatility and returns. Our analysis suggests that when studying other determinants of the cross-section of option returns, it is critical to first account for total volatility, and it indicates how volatility affects expected returns. In this sense our work is most closely related to that of Coval and Shumway (2001), who analyze moneyness as a determinant of different expected option returns using returns on index options.

Finally, our work is also relevant for an important literature on the sign of the volatility risk premium embedded in equity options. The consensus in the literature is that while the negative volatility risk premium is very large for the index, it is much smaller or nonexistent for equities. Most of the literature uses parametric models to characterize this risk premium, but some studies, such as Bakshi and Kapadia (2003b) have used the cross-section of delta-hedged option returns and arrive at the same conclusion. Our analysis in Section 6.1 shows that these empirical findings may be partly due to hedging errors, which generate a negative relation between volatility and delta-hedged call and put returns.

# 7 Volatility and the Time Series of Index Option Returns

Thus far we have used the cross-section of equity options to provide empirical evidence supporting Propositions 1 and 2. We now turn to the implications of our results for the extensive literature

[23] See Bakshi, Cao, and Zhong (2012) and Gourier (2015) for recent studies. Chaudhuri and Schroder (2015) study the shape of the stochastic discount factor based on equity options.

on the time series properties of index option returns.[24] In this section, we explore the time-series implications of Propositions 1 and 2 by studying the relation between monthly S&P 500 index option (SPX) returns and S&P 500 index volatility. Consistent with Proposition 1 and 2, we find that SPX call (put) options tend to have lower (higher) returns in the month following a high volatility month.

Propositions 1 and 2 characterize a general property of expected option returns: call (put) option returns decrease (increase) with underlying volatility. This property should hold in the time series of option returns as well as in the cross-section. We investigate the time-series implications of Propositions 1 and 2 by using index option returns to estimate the following time-series regression:

$$R_{t+1}^{i} = constant + \beta_1 VOL_t + \beta_2 Moneyness_t^{i} + \beta_3 R_t^{I} + \epsilon \tag{7.1}$$

where $R_{t+1}^{i}$ is the return on holding index option $i$ from month $t$ to month $t+1$, $R_t^I$ is the return on the S&P 500 in month $t$ and $VOL_t$ is the index volatility. Moneyness ($K/S$) is also included in the regression because previous studies (e.g., Coval and Shumway, 2001) have shown that moneyness is an important determinant of option returns. Here we consider four proxies for S&P 500 index volatility: 14-day realized volatility, 30-day realized volatility, 60-day realized volatility, and implied volatility. These volatilities are defined as in the cross-sectional analysis and are known in month $t$.

The slope coefficient estimate on volatility $\beta_1$ is the main object of interest. According to Propositions 1 and 2, we expect $\beta_1$ to be negative for SPX call options and positive for SPX put options.

Table 12 presents the coefficient estimates, t-statistics, and adjusted R-squares for the regressions in equation (7.1). Consistent with Propositions 1 and 2, the slope coefficient on index volatility is always negative (positive) for SPX call (put) options, regardless of the index volatility proxy. For example, column 2 of Panel A of Table 12 shows that when using 30-day realized volatility as the volatility proxy, the slope coefficient on index volatility is $-0.92$ for SPX calls and is highly significant with a t-statistic of $-3.78$. For a 1% increase in S&P 500 volatility, the return to holding an SPX call option over the next month is expected to decrease by 0.92%. In contrast, in column 2 of Panel B of Table 12, the slope coefficient on index volatility for SPX puts is 1.39 and it is also highly statistically significant.

[24] This literature includes the work by Jackwerth (2000), Coval and Shumway (2001), Bakshi and Kapadia (2003a), Bondarenko (2003), Jones (2006), Driessen and Maenhout (2007), Driessen, Maenhout, and Vilkov (2009), Santa-Clara and Saretto (2009), Broadie, Chernov, and Johannes (2009), Constantinides et al. (2009, 2011, 2013) and Buraschi, Trojani, and Vedolin (2014).

These results are based on the full sample that also contains in-the-money SPX options. However, Table 2 indicates that in-the-money SPX options are much less traded than their at-the-money and out-of-the-money counterparts. To ensure our results are not driven by illiquid in-the-money options, we repeat the regressions in (7.1) using only liquid options. Specifically, we only consider SPX calls with $0.98 \leq K/S \leq 1.10$ and SPX puts with $0.90 \leq K/S \leq 1.02$.

The regression results using only liquid options are presented in columns 5 through 8 in Table 12. Consistent with the results using the full sample, we find that the slope coefficient estimate on index volatility is always negative (positive) and statistically significant for SPX calls (puts) regardless of the volatility proxy. For example, when using 60-day realized volatility as a proxy, we find a slope coefficient of $-1.62$ for SPX calls and 1.58 for SPX puts, and both are highly significant with t-statistics of $-3.77$ and 2.98 respectively. These results confirm that our findings are not due to illiquid index options.

# 8 Conclusion

This paper analyzes the relation between expected option returns and underlying volatility. In the Black-Scholes-Merton or stochastic volatility model, the expected return on a call is a decreasing function of underlying volatility and the expected put option return is an increasing function of underlying volatility.

Our empirical results confirm this theoretical prediction. We conduct a cross-sectional test using stock options. We find that call (put) options on high volatility stocks tend to have lower (higher) returns over the next month. We also conduct a time-series test using index option returns. Following high volatility periods, index call (put) options tend to have lower (higher) returns over the next month. Our empirical findings are robust to different empirical implementation choices, such as different option samples, weighting methods, and volatility proxies. We also discuss results for straddles and we show that our results are consistent with existing findings on the relation between volatility and delta-hedged option returns.

Our findings are important for the expanding literature on equity option returns. Theory predicts that volatility is an important determinant of expected returns, and therefore volatility should be accounted for when empirically investigating other return determinants. Our findings also have important implications for other areas of finance research. Many financial instruments, such as credit default swaps, callable bonds, and levered equity, to name just a few, have embedded option features. Our theoretical results are also applicable to these assets and we plan to address this in future research. Our analysis can also be extended in several other ways. First, a natural question is if the relation between volatility and returns can be derived without asuming a parametric model. Second, it might be interesting to study the relationships in this paper using more structural rather than reduced-form asset pricing models. Third, Bakshi, Madan, and Panayotov (2010) and Christoffersen, Heston, and Jacobs (2013) propose variance dependent pricing kernels. In future work we plan to investigate the implications of those pricing kernels for the findings in this paper. Finally, the implications of our results for portfolio allocation need to be explored in more detail.

# Appendix A: Volatility and Instantaneous Option Returns

We show that expected instantaneous call option return is a decreasing function with respect to $\sigma$. We need to show that the elasticity $\frac{\partial O}{\partial S}\frac{S}{O}$, denoted by $EL$, is a decreasing function of $\sigma$. In the Black-Scholes-Merton model, we have

$$EL = \frac{\partial O}{\partial S}\frac{S}{O} = \frac{S_t N(d_1)}{S_t N(d_1) - e^{-r\tau} K N(d_2)}.$$

It follows that

$$\begin{aligned}
\frac{\partial EL}{\partial \sigma} &= \frac{S_t\psi(d_1)\frac{\partial d_1}{\partial\sigma}[S_tN(d_1) - e^{-r\tau}KN(d_2)] - S_tN(d_1)[S_t\psi(d_1)\frac{\partial d_1}{\partial\sigma} - e^{-r\tau}K\psi(d_2)\frac{\partial d_2}{\partial\sigma}]}{[S_tN(d_1) - e^{-r\tau}KN(d_2)]^2} \\
&= \frac{-S_t\psi(d_1)\frac{\partial d_1}{\partial\sigma}e^{-r\tau}KN(d_2) + S_tN(d_1)e^{-r\tau}K\psi(d_2)\frac{\partial d_2}{\partial\sigma}}{[S_tN(d_1) - e^{-r\tau}KN(d_2)]^2} \\
&= \frac{S_t\psi(d_1)\psi(d_2)e^{-r\tau}K[-\frac{\partial d_1}{\partial\sigma}\frac{N(d_2)}{\psi(d_2)} + \frac{N(d_1)}{\psi(d_1)}\frac{\partial d_2}{\partial\sigma}]}{[S_tN(d_1) - e^{-r\tau}KN(d_2)]^2}.
\end{aligned}$$

Clearly, the sign of $\frac{\partial EL}{\partial\sigma}$ will depend on $-\frac{N(d_2)}{\psi(d_2)}\frac{\partial d_1}{\partial\sigma} + \frac{N(d_1)}{\psi(d_1)}\frac{\partial d_2}{\partial\sigma}$, which we show below is always negative. To see this, using the fact that

$$\begin{aligned}
\frac{\partial d_1}{\partial\sigma} &= \sqrt{\tau} - \frac{d_1}{\sigma} \\
\frac{\partial d_2}{\partial\sigma} &= -\sqrt{\tau} - \frac{d_2}{\sigma},
\end{aligned}$$

we have

$$\begin{aligned}
-\frac{\partial d_1}{\partial\sigma}\frac{N(d_2)}{\psi(d_2)} + \frac{N(d_1)}{\psi(d_1)}\frac{\partial d_2}{\partial\sigma} &= -(\sqrt{\tau} - \frac{d_1}{\sigma})\frac{N(d_2)}{\psi(d_2)} + \frac{N(d_1)}{\psi(d_1)}(-\sqrt{\tau} - \frac{d_2}{\sigma}) \\
&= \frac{1}{\sigma}\{-(\sigma\sqrt{\tau} - d_1)\frac{N(d_2)}{\psi(d_2)} + \frac{N(d_1)}{\psi(d_1)}(-\sigma\sqrt{\tau} - d_2)\} \\
&= \frac{1}{\sigma}\{d_2\frac{N(d_2)}{\psi(d_2)} - d_1\frac{N(d_1)}{\psi(d_1)}\}.
\end{aligned}$$

Note that in the last step, we use the fact that $d_2 = d_1 - \sigma\sqrt{\tau}$. Finally, it can be shown that $x\frac{N(x)}{\psi(x)}$ is an increasing function in $x$, and therefore

$$d_1 > d_2 \Rightarrow d_2\frac{N(d_2)}{\psi(d_2)} - d_1\frac{N(d_1)}{\psi(d_1)} < 0.$$

# Appendix B: Proof of Proposition 2

The expected gross return of holding a put option to expiration in (2.4) can be rewritten using the Black-Scholes-Merton formula.

$$
\begin{aligned}
R_{put} &= \frac{E_t[\max(K - S_T, 0)]}{P_t(\tau, S_t, \sigma, K, r)} \\
&= \frac{\int^{z^*}(K - S_t e^{\mu\tau - \frac{1}{2}\sigma^2\tau + \sigma\sqrt{\tau}z})\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}dz}{P_t(\tau, S_t, \sigma, K, r)} \\
&= \frac{e^{\mu\tau}[e^{-\mu\tau}KN(-d_2^*) - S_tN(-d_1^*)]}{e^{-r\tau}KN(-d_2) - S_tN(-d_1)} \qquad \text{(B.1)}
\end{aligned}
$$

$$
d_1^* = \frac{\ln\frac{S_t}{K} + (\mu + \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}} \qquad d_2^* = \frac{\ln\frac{S_t}{K} + (\mu - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}
$$

$$
d_1 = \frac{\ln\frac{S_t}{K} + (r + \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}} \qquad d_2 = \frac{\ln\frac{S_t}{K} + (r - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}.
$$

Taking the derivative with respect to $\sigma$ in (B.1) yields:

$$
\begin{aligned}
\frac{\partial R_{put}}{\partial\sigma} &= \frac{e^{\mu\tau}\sqrt{\tau}S_t\psi(-d_1^*)[e^{-r\tau}KN(-d_2) - S_tN(-d_1)] - e^{\mu\tau}[e^{-\mu\tau}KN(-d_2^*) - S_tN(-d_1^*)]\sqrt{\tau}S_t\psi(-d_1)}{[e^{-r\tau}KN(-d_2) - S_tN(-d_1)]^2} \\
&= \frac{e^{\mu\tau}\sqrt{\tau}S_t\{\psi(-d_1^*)[e^{-r\tau}KN(-d_2) - S_tN(-d_1)] - \psi(-d_1)[e^{-\mu\tau}KN(-d_2^*) - S_tN(-d_1^*)]\}}{[e^{-r\tau}KN(-d_2) - S_tN(-d_1)]^2}
\end{aligned}
$$

where we use the fact that the Vega of a put option is $\sqrt{\tau}S_t\psi(-d_1)$. Clearly, the sign of $\frac{\partial R_{put}}{\partial\sigma}$ depends on $\psi(-d_1^*)[e^{-r\tau}KN(-d_2) - S_tN(-d_1)] - \psi(-d_1)[e^{-\mu\tau}KN(-d_2^*) - S_tN(-d_1^*)]$, which we denote by $B$. Next we show $B$ is positive. To see this,

$$
\begin{aligned}
B &= \psi(-d_1^*)[e^{-r\tau}KN(-d_2) - S_tN(-d_1)] - \psi(-d_1)[e^{-\mu\tau}KN(-d_2^*) - S_tN(-d_1^*)] \\
\frac{B}{\psi(-d_1^*)\psi(-d_1)} &= \frac{e^{-r\tau}KN(-d_2) - S_tN(-d_1)}{\psi(-d_1)} - \frac{e^{-\mu\tau}KN(-d_2^*) - S_tN(-d_1^*)}{\psi(-d_1^*)}.
\end{aligned}
$$

Using the fact that $e^{-r\tau}K\psi(-d_2) = S_t\psi(-d_1)$,

$$\begin{aligned}
\frac{B}{\psi(-d_1^*)\psi(-d_1)} &= \frac{\frac{S_t\psi(-d_1)}{\psi(-d_2)}N(-d_2) - S_tN(-d_1)}{\psi(-d_1)} - \frac{\frac{S_t\psi(-d_1^*)}{\psi(-d_2^*)}N(-d_2^*) - S_tN(-d_1^*)}{\psi(-d_1^*)} \\
&= S_t\{[\frac{N(-d_2)}{\psi(-d_2)} - \frac{N(-d_1)}{\psi(-d_1)}] - [\frac{N(-d_2^*)}{\psi(-d_2^*)} - \frac{N(-d_1^*)}{\psi(-d_1^*)}]\} \\
&= S_t\{[\frac{N(-d_1^*)}{\psi(-d_1^*)} - \frac{N(-d_2^*)}{\psi(-d_2^*)}] - [\frac{N(-d_1)}{\psi(-d_1)} - \frac{N(-d_2)}{\psi(-d_2)}]\}.
\end{aligned}$$

Because the expected rate of return on a risky asset exceeds the risk-free rate ($\mu > r$), we have $d_1^* > d_1$ and $d_2^* > d_2$. One can easily verify that $\frac{N(-d)}{\psi(-d)}$ is a decreasing and convex function in $d$. It follows that[25]

$$[\frac{N(-d_1^*)}{\psi(-d_1^*)} - \frac{N(-d_2^*)}{\psi(-d_2^*)}] - [\frac{N(-d_1)}{\psi(-d_1)} - \frac{N(-d_2)}{\psi(-d_2)}] > 0.$$

Therefore,

$$B > 0 \Rightarrow \frac{\partial R_{put}}{\partial \sigma} > 0.$$

# Appendix C: Holding-Period Expected Option Returns

We derive expected holding-period option returns in the Black-Scholes-Merton model. To save space, we only focus on call options. The analysis of put options proceeds along the same lines. To facilitate the notation, we consider an European call option at time 0 that matures at time $T$. By definition, the expected return of holding the call option from time 0 to time $h$ ($h < T$) is:

$$R_{call}^h = \frac{E_0\{S_hN(d_1') - e^{-r(T-h)}KN(d_2')\}}{S_0N(d_1) - e^{-rT}KN(d_2)}$$

where $S_hN(d_1') - e^{-r(T-h)}KN(d_2')$ is the future value of the option at time $h$, and

$$d_1' = \frac{\ln\frac{S_h}{K} + (r + \frac{1}{2}\sigma^2)(T-h)}{\sigma\sqrt{T-h}} \qquad d_2' = \frac{\ln\frac{S_h}{K} + (r - \frac{1}{2}\sigma^2)(T-h)}{\sigma\sqrt{T-h}}$$

$$d_1 = \frac{\ln\frac{S_0}{K} + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \qquad d_2 = \frac{\ln\frac{S_0}{K} + (r - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}.$$

[25] The second-order derivative of a decreasing and convex function is positive. Effectively $[\frac{N(-d_1^*)}{\psi(-d_1^*)} - \frac{N(-d_2^*)}{\psi(-d_2^*)}] - [\frac{N(-d_1)}{\psi(-d_1)} - \frac{N(-d_2)}{\psi(-d_2)}]$ is the second order derivative of $\frac{N(-d)}{\psi(-d)}$ with respect to $d$ and therefore it is positive.

The expected future value of the option at time $h$ can be split into two pieces:

$$\begin{aligned}
E_0\{S_h N(d_1') - e^{-r(T-h)} K N(d_2')\} &= \int_{-\infty}^{\infty} [S_0 e^{\mu h - \frac{1}{2}\sigma^2 h + \sigma\sqrt{h} z} N(d_1') - e^{-r(T-h)} K N(d_2')] \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \\
&= \int_{-\infty}^{\infty} S_0 e^{\mu h - \frac{1}{2}\sigma^2 h + \sigma\sqrt{h} z} N(d_1') \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \\
&\quad + \int_{-\infty}^{\infty} -e^{-r(T-h)} K N(d_2') \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz.
\end{aligned}$$

For the first integral, it can be shown that

$$\begin{aligned}
&\int_{-\infty}^{\infty} S_0 e^{\mu h - \frac{1}{2}\sigma^2 h + \sigma\sqrt{h} z} N(d_1') \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \\
= \; & S_0 e^{\mu h} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{(z-\sigma\sqrt{h})^2}{2}} N(\frac{\ln\frac{S_0}{K} + \mu h - \frac{1}{2}\sigma^2 h + \sigma\sqrt{h} z + (r + \frac{1}{2}\sigma^2)(T-h)}{\sigma\sqrt{T-h}}) dz. \quad \text{(C.1)}
\end{aligned}$$

Define a new variable $z^* = z - \sigma\sqrt{h}$. (C.1) becomes

$$S_0 e^{\mu h} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^{*2}}{2}} N(\frac{\ln\frac{S_0}{K} + (\mu - r)h + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T-h}} + \sqrt{\frac{h}{T-h}} z^*) dz^*. \tag{C.2}$$

Using (see Rubinstein 1984)

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^{*2}}{2}} N(A + Bz^*) = N(\frac{A}{\sqrt{1+B^2}}),$$

(C.2) can be further simplified as

$$S_0 e^{\mu h} N(\frac{\ln\frac{S_0}{K} + (\mu - r)h + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}). \tag{C.3}$$

Following the same steps, the second integral can be rewritten as

$$\int_{-\infty}^{\infty} -e^{-r(T-h)} K N(d_2') \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz = -e^{-r(T-h)} K N(\frac{\ln\frac{S_0}{K} + (\mu - r)h + (r - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}). \tag{C.4}$$

Putting (C.3) and (C.4) together, we obtain

$$R_{call}^h = \frac{S_0 e^{\mu h} N(\frac{\ln\frac{S_0}{K} + (\mu - r)h + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}) - e^{-r(T-h)} K N(\frac{\ln\frac{S_0}{K} + (\mu - r)h + (r - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}})}{S_0 N(d_1) - e^{-rT} K N(d_2)}.$$

This can be further simplified to

$$\begin{aligned} R^h_{call} &= \frac{e^{\mu h}[S_0 N(d_1^*) - e^{-[r+(\mu-r)HP]T} K N(d_2^*)]}{S_0 N(d_1) - e^{-rT} K N(d_2)} \\ d_1^* &= \frac{\ln \frac{S_0}{K} + [HP(\mu - r) + r + \frac{1}{2}\sigma^2]T}{\sigma\sqrt{T}} \\ d_2^* &= \frac{\ln \frac{S_0}{K} + [HP(\mu - r) + r - \frac{1}{2}\sigma^2]T}{\sigma\sqrt{T}} \end{aligned} \tag{C.5}$$

where $HP = h/T$.

# Appendix D: Expected Stock Returns and Expected Option Returns

We show that expected call (put) option returns increase (decrease) with expected stock returns: $\frac{\partial R_{call}}{\partial \mu} > 0$ and $\frac{\partial R_{put}}{\partial \mu} < 0$. First, recall from (2.12):

$$R_{call} = \frac{e^{\mu\tau}[S_t N(d_1^*) - e^{-\mu\tau} K N(d_2^*)]}{S_t N(d_1) - e^{-r\tau} K N(d_2)}$$

$$d_1^* = \frac{\ln \frac{S_t}{K} + (\mu + \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}} \qquad d_2^* = \frac{\ln \frac{S_t}{K} + (\mu - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}$$

$$d_1 = \frac{\ln \frac{S_t}{K} + (r + \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}} \qquad d_2 = \frac{\ln \frac{S_t}{K} + (r - \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}.$$

Taking the derivative with respect to $\mu$

$$\frac{\partial R_{call}}{\partial \mu} = \frac{\tau e^{\mu\tau}[S_t N(d_1^*) - e^{-\mu\tau} K N(d_2^*)] + e^{\mu\tau}[\tau e^{-\mu\tau} K N(d_2^*)]}{S_t N(d_1) - e^{-r\tau} K N(d_2)}$$

where $\psi$ is the probability density function of standard normal distribution. Note that we apply the fact that the Rho of a call option is $\tau e^{-\mu\tau} K N(d_2^*)$ in deriving the above equation. $\frac{\partial R_{call}}{\partial \mu}$ can be further simplified:

$$\begin{aligned} \frac{\partial R_{call}}{\partial \mu} &= \frac{\tau e^{\mu\tau}[S_t N(d_1^*) - e^{-\mu\tau} K N(d_2^*)] + \tau K N(d_2^*)}{S_t N(d_1) - e^{-r\tau} K N(d_2)} \\ &= \frac{\tau e^{\mu\tau} S_t N(d_1^*)}{S_t N(d_1) - e^{-r\tau} K N(d_2)} > 0. \end{aligned}$$

To see that the derivative is positive, notice that the denominator is just the price of call option which is always positive, and the numerator is obviously greater than zero.

Next we show that the expected put option return is a decreasing function of the expected stock return. Recall that the expected put option return is:

$$R_{put} = \frac{e^{\mu\tau}[e^{-\mu\tau}KN(-d_2^*) - S_tN(-d_1^*)]}{e^{-r\tau}KN(-d_2) - S_tN(-d_1)}$$

where $d_1^*$, $d_2^*$, $d_1$, and $d_2$ are defined the same as the above. Taking the derivative with respect to $\mu$ yields:

$$\begin{aligned}\frac{\partial R_{put}}{\partial \mu} &= \frac{\tau e^{\mu\tau}[e^{-\mu\tau}KN(-d_2^*) - S_tN(-d_1^*)] + e^{\mu\tau}[-\tau e^{-\mu\tau}KN(-d_2^*)]}{e^{-r\tau}KN(-d_2) - S_tN(-d_1)} \\ &= \frac{-\tau e^{\mu\tau}S_tN(-d_1^*)}{e^{-r\tau}KN(-d_2) - S_tN(-d_1)} < 0.\end{aligned}$$

Note the denominator is the price of put option which is always positive, and therefore the ratio itself is negative.

# Appendix E: Expected Straddle Returns

We study the relation between expected straddle returns and the underlying volatility. The expected gross return on a straddle is defined as

$$\begin{aligned}R_{straddle} &= \frac{E_t[\max(S_T - K, 0)] + E_t[\max(K - S_T, 0)]}{C_t(\tau, S_t, \sigma, K, r) + P_t(\tau, S_t, \sigma, K, r)} \\ &= \frac{[S_tN(d_1^*) - e^{-\mu\tau}KN(d_2^*)]e^{\mu\tau} + [e^{-\mu\tau}KN(-d_2^*) - S_tN(-d_1^*)]e^{\mu\tau}}{S_tN(d_1) - e^{-r\tau}KN(d_2) + e^{-r\tau}KN(-d_2) - S_tN(-d_1)}.\end{aligned}$$

We investigate the impact of volatility on expected straddle returns by taking the derivative of $R_{straddle}$ with respect to $\sigma$. It follows that

$$\begin{aligned}\frac{\partial R_{straddle}}{\partial \sigma} &= \frac{2e^{\mu\tau}\sqrt{\tau}S_t\psi(d_1^*)A - 2e^{\mu\tau}\sqrt{\tau}S_t\psi(d_1)B}{[S_tN(d_1) - e^{-r\tau}KN(d_2) + e^{-r\tau}KN(-d_2) - S_tN(-d_1)]^2} \\ &= \frac{2e^{\mu\tau}\sqrt{\tau}S_t\{\psi(d_1^*)A - \psi(d_1)B\}}{[S_tN(d_1) - e^{-r\tau}KN(d_2) + e^{-r\tau}KN(-d_2) - S_tN(-d_1)]^2}\end{aligned}$$

where $A = S_tN(d_1) - e^{-r\tau}KN(d_2) + e^{-r\tau}KN(-d_2) - S_tN(-d_1)$ and $B = S_tN(d_1^*) - e^{-\mu\tau}KN(d_2^*) + e^{-\mu\tau}KN(-d_2^*) - S_tN(-d_1^*)$. It is clear that the sign of $\frac{\partial R_{straddle}}{\partial \sigma}$ is determined by $\psi(d_1^*)A - \psi(d_1)B$.

This term can be positive or negative depending on underlying parameters.

We now show that $d_2 > 0$ implies $\psi(d_1^*)A - \psi(d_1)B < 0$ and therefore $\frac{\partial R_{straddle}}{\partial \sigma} < 0$. First recall from previous analysis $d_1^* > d_1 > d_2$. We then have

$$d_2 > 0 \Rightarrow 0 < \psi(d_1^*) < \psi(d_1). \tag{E.1}$$

Moreover, note that

$$\frac{\partial A}{\partial r} = \tau e^{-r\tau} K[N(d_2) - N(-d_2)]$$

and therefore,

$$d_2 > 0 \Rightarrow \frac{\partial A}{\partial r} > 0$$

which further implies

$$0 < A < B \tag{E.2}$$

by noting that $B$ is obtained by replacing $r$ with $\mu$ in $A$. Putting together (E.1) and (E.2),

$$d_2 > 0 \Rightarrow \psi(d_1^*)A - \psi(d_1)B < 0 \Rightarrow \frac{\partial R_{straddle}}{\partial \sigma} < 0.$$

# Appendix F: Expected Option Returns in the Heston Model

We derive the expected return of holding a call option to expiration in the Heston (1993) stochastic volatility model. The Heston (1993) model assumes that the asset price and its spot variance obey the following dynamics under the physical measure P

$$\begin{aligned} dS_t &= \mu S_t dt + S_t \sqrt{V_t} dZ_1^P \\ dV_t &= \kappa(\theta - V_t)dt + \sigma \sqrt{V_t} dZ_2^P \end{aligned}$$

where $\mu$ is the drift of the stock price, $\theta$ is the long run mean of the stock variance, $\kappa$ is the rate of mean reversion, $\sigma$ is the volatility of volatility, and $Z_1$ and $Z_2$ are two correlated Brownian motions with $E[dZ_1 dZ_2] = \rho dt$. The dynamics under the risk-neutral measure Q are

$$\begin{aligned} dS_t &= r S_t dt + S_t \sqrt{V_t} dZ_1^Q \\ dV_t &= [\kappa(\theta - V_t) - \lambda V_t]dt + \sigma \sqrt{V_t} dZ_2^Q \end{aligned}$$

where $r$ is the risk-free rate and $\lambda$ is the market price of volatility risk. Again we consider the expected return of holding a call option to expiration:

$$R_{Call}^{Heston}(S_t, V_t, \tau) = \frac{E_t[\max(S_T - K, 0)]}{C_t(t, T, S_t, V_t))} = \frac{E_t^P[\max(S_T - K, 0)]}{E_t^Q[e^{-r\tau}\max(S_T - K, 0)]}.$$

Heston (1993) provides a closed-form solution to an European call option, up to a univariate numerical integral:

$$C(t, T, S_t, V_t) = E_t^Q[e^{-r\tau}\max(S_T - K, 0)] = S_t P_1 - e^{-r\tau} K P_2 \tag{F.1}$$

where $P_1$ and $P_2$ are given by[26]

$$P_j = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}(\frac{e^{-i\phi \ln K} f_j(x, V, \tau; \phi)}{i\phi})d\phi \tag{F.2}$$

$$f_j(x, V, \tau; \phi) = e^{C(\tau;\phi)+D(\tau;\phi)V+i\phi x}$$

$$C(\tau; \phi) = r\phi i\tau + \frac{a}{\sigma^2}\{(b_j - \rho\sigma\phi i + d)\tau - 2\ln[\frac{1 - ge^{d\tau}}{1 - g}]\}$$

$$D(\tau; \phi) = \frac{b_j - \rho\sigma\phi i + d}{\sigma^2}[\frac{1 - e^{d\tau}}{1 - ge^{d\tau}}]$$

$$g = \frac{b_j - \rho\sigma\phi i + d}{b_j - \rho\sigma\phi i - d}$$

$$d = \sqrt{(\rho\sigma\phi i - b_j)^2 - \sigma^2(2u_j\phi i - \phi^2)}$$

$$u_1 = \frac{1}{2}, u_2 = -\frac{1}{2}, a = \kappa\theta, b_1 = \kappa + \lambda - \rho\sigma, b_2 = \kappa + \lambda.$$

By analogy, it can be shown that expected call option payoff at expiration is

$$E_t^P[\max(S_T - K), 0] = e^{\mu\tau}[S_t P_1^* - e^{-\mu\tau} K P_2^*] \tag{F.3}$$

where

$$P_j^* = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}(\frac{e^{-i\phi \ln K} f_j^*(x, V, \tau; \phi)}{i\phi})d\phi \tag{F.4}$$

$$f_j^*(x, V, \tau; \phi) = e^{C(\tau;\phi)+D(\tau;\phi)V+i\phi x}$$

[26] Note that $x = \ln S$.

$$C(\tau;\phi) = \mu\phi i\tau + \frac{a}{\sigma^2}\{(b_j - \rho\sigma\phi i + d)\tau - 2\ln[\frac{1-ge^{d\tau}}{1-g}]\}$$

$$D(\tau;\phi) = \frac{b_j - \rho\sigma\phi i + d}{\sigma^2}[\frac{1-e^{d\tau}}{1-ge^{d\tau}}]$$

$$g = \frac{b_j - \rho\sigma\phi i + d}{b_j - \rho\sigma\phi i - d}$$

$$d = \sqrt{(\rho\sigma\phi i - b_j)^2 - \sigma^2(2u_j\phi i - \phi^2)}$$

$$u_1 = \frac{1}{2}, u_2 = -\frac{1}{2}, a = \kappa\theta, b_1 = \kappa - \rho\sigma, b_2 = \kappa.$$

Putting (F.1) and (F.3) together, the analytical expected holding-to-maturity call option return in Heston model is

$$R_{Call}^{Heston}(S_t, V_t, \tau) = \frac{e^{\mu\tau}[S_t P_1^* - e^{-\mu\tau} K P_2^*]}{S_t P_1 - e^{-r\tau} K P_2}. \tag{F.5}$$

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Table 1: Summary Statistics for Equity Options

We report averages by moneyness category of monthly equity option returns (return), the underlying stock's realized volatility over the preceding month (30-day realized vol), option implied volatility (implied vol), option volume (volume) and the option Greeks. Panel A reports on call options and Panel B on put options. We compute monthly option returns using the midpoint of bid and ask quotes. Realized volatility is calculated as the standard deviation of the logarithm of daily returns over the preceding month. The sample consists of options that are at-the-money ($0.95 \leq K/S \leq 1.05$) and approximately one month from expiration. The sample period is from January 1996 to July 2013.

| Moneyness $K/S$ | $[0.95-0.97]$ | $(0.97-0.99]$ | $(0.99-1.01]$ | $(1.01-1.03]$ | $(1.03-1.05]$ |
|---|---|---|---|---|---|
| Panel A: Call Options | | | | | |
| Return | 0.054 | 0.080 | 0.111 | 0.119 | 0.100 |
| 30-day realized vol | 47.06% | 45.57% | 44.70% | 44.23% | 44.97% |
| Implied vol | 49.03% | 46.94% | 45.49% | 44.90% | 45.44% |
| Volume | 232 | 306 | 385 | 430 | 396 |
| Open interest | 1846 | 1855 | 1798 | 1897 | 1885 |
| Delta | 0.68 | 0.61 | 0.53 | 0.45 | 0.38 |
| Gamma | 0.11 | 0.12 | 0.14 | 0.13 | 0.12 |
| Vega | 4.41 | 4.81 | 4.95 | 4.89 | 4.52 |
| | | | | | |
| Panel B: Put Options | | | | | |
| Return | -0.137 | -0.121 | -0.100 | -0.104 | -0.087 |
| 30-day realized vol | 45.86% | 44.88% | 45.51% | 46.19% | 47.62% |
| Implied vol | 48.97% | 47.29% | 47.01% | 47.24% | 48.25% |
| Volume | 318 | 359 | 340 | 278 | 207 |
| Open interest | 1875 | 1841 | 1672 | 1670 | 1563 |
| Delta | -0.33 | -0.39 | -0.47 | -0.55 | -0.61 |
| Gamma | 0.10 | 0.11 | 0.13 | 0.12 | 0.11 |
| Vega | 4.69 | 5.15 | 5.27 | 5.25 | 4.87 |

Table 2: Summary Statistics for S&P 500 Index Options

We report averages of monthly S&P 500 index option returns (return), implied volatility (implied vol), option volume (volume), and option Greeks by moneyness. Panel A reports on call options and Panel B reports on put options. We compute the monthly option return using the midpoint of the bid and ask quotes. The sample consists of S&P 500 index options (SPX) with moneyness $0.90 \leq K/S \leq 1.10$ and one-month maturity. The sample period is from January 1996 to July 2013.

| Moneyness $K/S$ | [0.90−0.94] | (0.94−0.98] | (0.98−1.02] | (1.02−1.06] | (1.06−1.10] |
|---|---|---|---|---|---|
| Panel A: SPX Call Options | | | | | |
| Return | 0.027 | 0.057 | 0.060 | -0.112 | -0.617 |
| Implied vol | 27.30% | 22.75% | 19.68% | 17.42% | 17.28% |
| Volume | 251 | 306 | 2029 | 2867 | 2156 |
| Open interest | 9679 | 11770 | 15236 | 15388 | 14807 |
| Delta | 0.88 | 0.76 | 0.51 | 0.20 | 0.06 |
| Gamma | 0.002 | 0.005 | 0.007 | 0.005 | 0.002 |
| Vega | 60.32 | 93.12 | 119.86 | 80.66 | 32.99 |
| Panel B: SPX Put Options | | | | | |
| Return | -0.540 | -0.406 | -0.224 | -0.133 | -0.171 |
| Implied vol | 26.56% | 22.87% | 19.66% | 18.20% | 22.68% |
| Volume | 3699 | 2662 | 2619 | 391 | 338 |
| Open interest | 19604 | 18649 | 14674 | 8992 | 12322 |
| Delta | -0.11 | -0.23 | -0.48 | -0.75 | -0.88 |
| Gamma | 0.002 | 0.005 | 0.007 | 0.006 | 0.003 |
| Vega | 55.13 | 90.56 | 119.80 | 93.61 | 53.04 |

Table 3: Option Portfolio Returns Sorted on Underlying Volatility

We report average equal-weighted monthly returns for option portfolios sorted on 30-day realized volatility, as well as the return differences between the two extreme portfolios. Panel A reports on call options and Panel B on put options. Panel C reports results for option returns based on ask prices rather than the midpoint of bid and ask quotes. Every month, all available one-month at-the-money options are sorted into five quintile portfolios according to their 30-day realized volatility. Portfolio Low (High) contains options with the lowest (highest) underlying volatilities. The sample period is from January 1996 to July 2013. Newey-West t-statistics using four lags are reported in parentheses. Statistical significance at the 10%, 5%, and 1% level is denoted by *, **, and *** respectively.

| | | | | | | |
|---|---|---|---|---|---|---|
| Panel A: Call Option Portfolios | | | | | | |
| | Low | 2 | 3 | 4 | High | H-L |
| $0.95 \leq K/S \leq 1.05$ | 0.147 | 0.128 | 0.111 | 0.084 | 0.009 | -0.138*** |
| | | | | | | (-3.422) |
| $0.975 \leq K/S \leq 1.025$ | 0.155 | 0.145 | 0.120 | 0.094 | 0.017 | -0.138*** |
| | | | | | | (-3.496) |
| Panel B: Put Option Portfolios | | | | | | |
| | Low | 2 | 3 | 4 | High | H-L |
| $0.95 \leq K/S \leq 1.05$ | -0.146 | -0.153 | -0.109 | -0.077 | -0.075 | 0.071** |
| | | | | | | (2.004) |
| $0.975 \leq K/S \leq 1.025$ | -0.145 | -0.157 | -0.101 | -0.065 | -0.068 | 0.077** |
| | | | | | | (2.081) |
| Panel C: Using Ask Prices | | | | | | |
| | Low | 2 | 3 | 4 | High | H-L |
| Call Option Portfolios | 0.048 | 0.045 | 0.033 | 0.012 | -0.060 | -0.108*** |
| | | | | | | (-2.942) |
| Put Option Portfolios | -0.209 | -0.209 | -0.165 | -0.133 | -0.133 | 0.076** |
| | | | | | | (2.302) |

Table 4: Option Portfolio Returns Double-Sorted on Expected Stock Return and Underlying Volatility

We report average equal-weighted monthly returns on option portfolios sorted on expected stock return ($\mu$) and 30-day realized volatility. Panel A reports on call options and Panel B on put options. Every month, all available one-month at-the-money options are first ranked into five quintile portfolios according to the underlying stocks' expected returns. Then, within each $\mu$ quintile, options are further sorted into five portfolios based on 30-day realized volatility. Portfolio Low (High) contains options with the lowest (highest) underlying volatility. Following Boyer and Vorkink (2014), the expected stock return is estimated as the simple average of daily returns over the past six month preceding the portfolio formation date. The sample period is from January 1996 to July 2013. Newey-West t-statistics with four lags are reported in parentheses. Statistical significance at the 10%, 5% and 1% level is denoted by *, **, and *** respectively.

| Panel A: Call Options | | Low | 3 | 3 | 4 | High | H-L |
|---|---|---|---|---|---|---|---|
| | 1 | 0.246 | 0.151 | 0.075 | 0.033 | 0.001 | -0.245*** |
| | | | | | | | (-5.889) |
| | 2 | 0.190 | 0.148 | 0.117 | 0.085 | -0.006 | -0.195*** |
| $\mu$ Quintiles | | | | | | | (-3.628) |
| | 3 | 0.146 | 0.170 | 0.125 | 0.082 | 0.021 | -0.125*** |
| | | | | | | | (-2.769) |
| | 4 | 0.131 | 0.122 | 0.136 | 0.094 | 0.018 | -0.112*** |
| | | | | | | | (-2.854) |
| | 5 | 0.154 | 0.106 | 0.101 | 0.066 | 0.038 | -0.116*** |
| | | | | | | | (-2.823) |
| | | | | | | | |
| Panel B: Put Options | | Low | 2 | 3 | 4 | High | H-L |
| | 1 | -0.113 | -0.079 | -0.067 | -0.028 | -0.044 | 0.069* |
| | | | | | | | (1.799) |
| | 2 | -0.162 | -0.136 | -0.117 | -0.102 | -0.056 | 0.107** |
| | | | | | | | (2.092) |
| $\mu$ Quintiles | 3 | -0.153 | -0.187 | -0.162 | -0.095 | -0.078 | 0.074* |
| | | | | | | | (1.762) |
| | 4 | -0.154 | -0.158 | -0.136 | -0.116 | -0.133 | 0.021 |
| | | | | | | | (0.450) |
| | 5 | -0.182 | -0.095 | -0.132 | -0.102 | -0.079 | 0.103*** |
| | | | | | | | (3.165) |

Table 5: Controlling for Expected Stock Returns Using the CAPM

Panel A reports average equal-weighted monthly returns on option portfolios sorted on market beta and 30-day realized volatility. Every month, all available one-month at-the-money options are first ranked into five quintile portfolios according to the underlying stocks' CAPM betas. Then, within each beta quintile, options are further sorted into five portfolios based on 30-day realized volatility. Portfolio Low (High) contains options with the lowest (highest) underlying volatility. The CAPM beta is estimated using daily returns over the past 30 days preceding the portfolio formation date. Panel B reports average equal-weighted returns on option portfolios sorted on stock idiosyncratic volatility. Idiosyncratic volatility is estimated from the market model (CAPM) using daily returns over the past 30 days preceding the portfolio formation date. The sample period is from January 1996 to July 2013. Newey-West t-statistics with four lags are reported in parentheses. Statistical significance at the 10%, 5% and 1% level is denoted by *, **, and *** respectively.

| Panel A: Double Sorts on Beta and Volatility | | | | | | | |
|---|---|---|---|---|---|---|---|
| Beta\Vol | | Low | 2 | 3 | 4 | High | H-L |
| | 1 | 0.156 | 0.126 | 0.094 | 0.067 | -0.01 | -0.165*** |
| | | | | | | | (-3.384) |
| | 2 | 0.162 | 0.165 | 0.15 | 0.135 | 0.061 | -0.101** |
| | | | | | | | (-2.082) |
| Call | 3 | 0.149 | 0.194 | 0.147 | 0.112 | 0.05 | -0.099* |
| | | | | | | | (-1.969) |
| | 4 | 0.113 | 0.133 | 0.107 | 0.106 | 0.031 | -0.082** |
| | | | | | | | (-2.024) |
| | 5 | 0.09 | 0.076 | 0.104 | 0.022 | 0.005 | -0.085** |
| | | | | | | | (-2.115) |
| Beta\Vol | | Low | 2 | 3 | 4 | High | H-L |
| | 1 | -0.15 | -0.095 | -0.132 | -0.126 | -0.121 | 0.029 |
| | | | | | | | (0.618) |
| | 2 | -0.149 | -0.165 | -0.156 | -0.101 | -0.107 | 0.041 |
| | | | | | | | (0.896) |
| Put | 3 | -0.147 | -0.198 | -0.112 | -0.069 | -0.065 | 0.082** |
| | | | | | | | (1.997) |
| | 4 | -0.14 | -0.122 | -0.14 | -0.071 | -0.047 | 0.093** |
| | | | | | | | (2.087) |
| | 5 | -0.133 | -0.106 | -0.085 | -0.067 | -0.065 | 0.068* |
| | | | | | | | (1.957) |

| Panel B: Sorts on Idiosyncratic Volatility | | | | | | |
|---|---|---|---|---|---|---|
| | Low | 2 | 3 | 4 | High | H-L |
| Call | 0.156 | 0.13 | 0.119 | 0.083 | 0.021 | -0.133*** |
| | | | | | | (-3.245) |
| Put | -0.157 | -0.151 | -0.119 | -0.069 | -0.077 | 0.080** |
| | | | | | | (2.271) |

Table 6: Fama-MacBeth Regressions

We report results for the Fama-MacBeth regressions $R^i_{t+1} = \gamma_{0,t} + \gamma_{1,t} VOL^i_t + \Phi_t Z^i_t + \epsilon$, where $R^i_{t+1}$ is the option return, $VOL^i_t$ is the underlying stock volatility, and $Z^i_t$ is a vector of control variables that includes the stock's beta (beta), firm size (size), book-to-market (btm), momentum (mom), stock return reversal (reversal), option skew, volatility risk premium (vrp), the slope of the implied volatility term structure (slope), and option characteristics such as moneyness, Delta, Vega, Gamma, and option beta. Newey-West t-statistics with four lags are reported in parentheses. The sample consists of one-month at-the-money options. The sample period is from January 1996 to July 2013. Newey-West t-statistics with four lags are reported in parentheses. Statistical significance at the 10%, 5%, and 1% level is denoted by *, **, and *** respectively.

| | Calls | | | Puts | | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Vol | **-0.239*** ** | **-0.277*** ** | **-0.389** ** | **0.117** ** | **0.125*** ** | **0.584*** ** |
| | (-4.192) | (-5.293) | (-2.241) | (2.552) | (2.725) | (2.911) |
| Beta | | -0.004 | 0.045*** | | 0.000 | -0.034* |
| | | (-0.284) | (2.714) | | (-0.002) | (-1.789) |
| Size | | 0.000 | 0.001 | | 0.000 | -0.002** |
| | | (0.185) | (0.829) | | (-0.234) | (-2.285) |
| Btm | | 0.002 | 0.057* | | 0.005 | -0.060* |
| | | (0.938) | (1.754) | | (0.798) | (-1.832) |
| Mom | | 0.026 | 0.020 | | -0.023 | -0.013 |
| | | (0.971) | (0.824) | | (-1.252) | (-0.666) |
| Reversal | | | -0.188* | | | -0.162* |
| | | | (-1.940) | | | (-1.828) |
| Option skew | | | 0.065 | | | 0.474** |
| | | | (0.256) | | | (2.133) |
| Vrp | | | -0.059 | | | 0.454** |
| | | | (-0.327) | | | (1.982) |
| Slope | | | 0.685*** | | | 0.648** |
| | | | (3.270) | | | (2.580) |
| Moneyness | | | -0.090 | | | -0.920 |
| | | | (-0.090) | | | (-0.944) |
| Delta | | | -0.069 | | | -0.381 |
| | | | (-0.160) | | | (-1.000) |
| Vega | | | -0.021** | | | 0.015 |
| | | | (-2.347) | | | (1.379) |
| Gamma | | | 0.011 | | | 0.156 |
| | | | (0.022) | | | (0.233) |
| Option beta | | | -0.002 | | | -0.012 |
| | | | (-0.197) | | | (-0.847) |

Table 7: Expected Option Returns in the Heston Model

We report expected monthly option returns in the Heston (1993) stochastic volatility model. Panel B (C) reports expected returns on at-the-money call (put) options for different levels of the current stock variance ($V_t$). Panel D reports expected returns on at-the-money straddle for different levels of the volatility risk premium ($\lambda$). The computations are based on the model parameters reported in Broadie, Chernov, and Johannes (2009), which are calibrated from historical S&P 500 index return data. These parameters are reported in Panel A. For simplicity, the dividend yield is set to zero. We set $\lambda$ equal to 0 in Panels B and C, and we set $V_t$ equal to 0.0225 in Panel D.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Panel A: Parameters | | | $r$ | $\sqrt{\theta}$ | $\kappa$ | $\sigma$ | $\rho$ | $T-t$ | | |
| | | | 0.045 | 0.15 | 5.33 | 0.14 | -0.52 | 1/12 | | |
| Panel B: Call Options | | | | | | $V_t$ | | | | |
| | | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
| | 8% | 0.118 | 0.094 | 0.080 | 0.072 | 0.066 | 0.061 | 0.057 | 0.054 | 0.051 |
| $\mu$ | 12% | 0.257 | 0.202 | 0.172 | 0.153 | 0.139 | 0.128 | 0.120 | 0.113 | 0.107 |
| | 16% | 0.405 | 0.316 | 0.268 | 0.237 | 0.215 | 0.198 | 0.185 | 0.174 | 0.165 |
| Panel C: Put Options | | | | | | $V_t$ | | | | |
| | | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
| | 8% | -0.104 | -0.081 | -0.068 | -0.060 | -0.054 | -0.049 | -0.046 | -0.043 | -0.040 |
| $\mu$ | 12% | -0.217 | -0.172 | -0.146 | -0.130 | -0.117 | -0.108 | -0.100 | -0.094 | -0.089 |
| | 16% | -0.320 | -0.256 | -0.220 | -0.195 | -0.178 | -0.164 | -0.153 | -0.144 | -0.136 |
| Panel D: Straddles | | | | | | $\lambda$ | | | | |
| | | -0.5 | -0.4 | -0.3 | -0.2 | -0.1 | 0.0 | 0.1 | 0.2 | 0.3 |
| $\mu$ | 12% | 0.022 | 0.024 | 0.026 | 0.028 | 0.030 | 0.031 | 0.033 | 0.035 | 0.037 |

Table 8: Option Portfolio Returns Sorted on Alternative Volatility Measures

We report equal-weighted monthly option portfolio returns sorted on different measures of underlying volatility, as well as the return differences between the two extreme portfolios. Panel A reports on call options and Panel B reports on put options. We consider four volatility measures: realized volatility over the previous 14 days, realized volatility over the previous 60 days, realized volatility over the previous year, option-implied volatility as well as a measure of conditional volatility from an AR(1) model estimated on monthly realized volatilities. Every month, all available options are ranked into five quintile portfolios based on underlying volatility. Portfolio Low (High) contains options with the lowest (highest) underlying volatilities. The sample consists of one-month at-the-money options. The sample period is from January 1996 to July 2013. Newey-West t-statistics with four lags are reported in parentheses. Statistical significance at the 10%, 5%, and 1% level is denoted by *, **, and *** respectively.

| | Low | 2 | 3 | 4 | High | H-L |
|---|---|---|---|---|---|---|
| Panel A: Call Options | | | | | | |
| 14-day realized vol | 0.146 | 0.122 | 0.114 | 0.081 | 0.016 | -0.130*** |
| | | | | | | (-3.539) |
| 60-day realized vol | 0.155 | 0.109 | 0.115 | 0.086 | 0.014 | -0.141*** |
| | | | | | | (-3.437) |
| 365-day realized vol | 0.130 | 0.104 | 0.117 | 0.084 | 0.044 | -0.086* |
| | | | | | | (-1.805) |
| Implied vol | 0.156 | 0.117 | 0.134 | 0.081 | -0.010 | -0.166*** |
| | | | | | | (-3.598) |
| AR(1) vol | 0.117 | 0.107 | 0.112 | 0.080 | 0.018 | -0.099** |
| | | | | | | (-2.142) |
| | | | | | | |
| Panel B: Put Options | | | | | | |
| 14-day realized vol | -0.146 | -0.139 | -0.103 | -0.086 | -0.087 | 0.059* |
| | | | | | | (1.765) |
| 60-day realized vol | -0.157 | -0.151 | -0.109 | -0.084 | -0.059 | 0.098** |
| | | | | | | (2.488) |
| 365-day realized vol | -0.170 | -0.144 | -0.120 | -0.071 | -0.053 | 0.117*** |
| | | | | | | (2.817) |
| Implied vol | -0.130 | -0.143 | -0.118 | -0.087 | -0.083 | 0.047 |
| | | | | | | (1.128) |
| AR(1) vol | -0.171 | -0.154 | -0.129 | -0.077 | -0.014 | 0.157*** |
| | | | | | | (3.868) |

Table 9: Option Portfolio Returns for Alternative Option Samples

We report equal-weighted monthly option portfolio returns sorted on 30-day realized volatility, as well as the return differences between the two extreme portfolios. Different option samples are used: two-month at-the-money (ATM) options, one-month in-the-money (ITM) options, two-month ITM options, one-month out-of-the-money (OTM) options, and two-month OTM options. ATM options are defined by moneyness $0.95 \leq K/S \leq 1.05$, ITM options are defined by moneyness $0.80 \leq K/S < 0.95$ for calls and $1.05 < K/S \leq 1.20$ for puts, and OTM options are defined by moneyness $1.05 < K/S \leq 1.20$ for calls and $0.80 \leq K/S < 0.95$ for puts. Returns are reported as raw returns for the relevant horizons. The sample period is from January 1996 to July 2013. Newey-West t-statistics with four lags are reported in parentheses. Statistical significance at the 10%, 5%, and 1% level is denoted by *, **, and *** respectively.

| | Low | 2 | 3 | 4 | High | H-L |
|---|---|---|---|---|---|---|
| Panel A: Call Options | | | | | | |
| Two-month ATM | 0.144 | 0.135 | 0.112 | 0.035 | -0.027 | -0.171*** |
| | | | | | | (-3.037) |
| One-month ITM | 0.053 | 0.060 | 0.042 | 0.026 | -0.025 | -0.078*** |
| | | | | | | (-3.678) |
| Two-month ITM | 0.089 | 0.084 | 0.068 | 0.027 | -0.067 | -0.156*** |
| | | | | | | (-5.224) |
| One-month OTM | 0.055 | 0.049 | 0.077 | 0.048 | -0.066 | -0.121** |
| | | | | | | (-2.214) |
| Two-month OTM | 0.132 | 0.088 | 0.098 | 0.022 | -0.054 | -0.186** |
| | | | | | | (-2.361) |
| Panel B: Put Options | | | | | | |
| Two-month ATM | -0.207 | -0.149 | -0.118 | -0.079 | -0.056 | 0.151*** |
| | | | | | | (3.203) |
| One-month ITM | -0.091 | -0.069 | -0.052 | -0.043 | -0.034 | 0.057*** |
| | | | | | | (2.933) |
| Two-month ITM | -0.127 | -0.090 | -0.055 | -0.048 | -0.023 | 0.105*** |
| | | | | | | (3.625) |
| One-month OTM | -0.309 | -0.217 | -0.193 | -0.090 | -0.131 | 0.178*** |
| | | | | | | (2.759) |
| Two-month OTM | -0.276 | -0.197 | -0.188 | -0.118 | -0.099 | 0.177** |
| | | | | | | (2.037) |

Table 10: Delta-Hedged Option Returns

Panel A reports average monthly returns (in percent) for delta-hedged option portfolios sorted on underlying volatility. We use the sample from Table 3. Every month options are sorted into five quintile portfolios based on underlying volatility and the delta-hedged option return is computed over the following month according to Goyal and Saretto (2009) with no rebalancing. We report both equal-weighted (EW) and volume-weighted (VW) portfolio returns. The sample period is from January 1996 to July 2013. Newey-West t-statistics with four lags are reported in parentheses. Statistical significance at the 10%, 5%, and 1% level is denoted by *, **, and *** respectively. Panel B reports expected static delta-hedged returns in the Black-Scholes-Merton model calculated according to equation (6.4) for a hypothetical 1-month call option (returns are stated in percent). We consider different values of the moneyness $K/S$, the expected stock return $\mu$, and the underlying volatility $\sigma$. The annualized interest rate is assumed to be 3%.

Panel A: Delta-Hedged Option Returns

| | | Low | 2 | 3 | 4 | High | H-L |
|---|---|---|---|---|---|---|---|
| Calls | EW | -0.292 | -0.325 | -0.356 | -0.231 | -0.721 | -0.429** |
| | | | | | | | (-2.013) |
| | VW | -0.180 | -0.359 | -0.380 | -0.205 | -0.863 | -0.683** |
| | | | | | | | (-2.396) |
| Puts | EW | 0.027 | -0.067 | 0.046 | 0.003 | -0.484 | -0.511*** |
| | | | | | | | (-2.790) |
| | VW | 0.007 | -0.296 | 0.001 | 0.004 | -0.616 | -0.623** |
| | | | | | | | (-2.260) |

Panel B: Delta-Hedged Option Returns in the Black-Scholes-Merton Model

| | $\mu \backslash \sigma$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
|---|---|---|---|---|---|---|---|---|---|---|
| K/S=100/100 | 0.05 | 0.191 | 0.096 | 0.064 | 0.048 | 0.038 | 0.032 | 0.027 | 0.024 | 0.021 |
| | 0.07 | 0.764 | 0.384 | 0.256 | 0.192 | 0.154 | 0.128 | 0.109 | 0.096 | 0.085 |
| | 0.09 | 1.714 | 0.864 | 0.577 | 0.432 | 0.346 | 0.288 | 0.246 | 0.215 | 0.191 |
| | 0.11 | 3.039 | 1.536 | 1.026 | 0.769 | 0.615 | 0.512 | 0.438 | 0.383 | 0.340 |
| | 0.13 | 4.734 | 2.400 | 1.603 | 1.203 | 0.962 | 0.801 | 0.685 | 0.599 | 0.531 |
| | $\mu \backslash \sigma$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
| K/S=100/95 | 0.05 | 0.046 | 0.066 | 0.053 | 0.043 | 0.035 | 0.030 | 0.026 | 0.023 | 0.020 |
| | 0.07 | 0.192 | 0.267 | 0.215 | 0.172 | 0.142 | 0.120 | 0.104 | 0.091 | 0.081 |
| | 0.09 | 0.446 | 0.605 | 0.485 | 0.388 | 0.320 | 0.270 | 0.234 | 0.206 | 0.183 |
| | 0.11 | 0.820 | 1.085 | 0.866 | 0.692 | 0.569 | 0.482 | 0.416 | 0.366 | 0.326 |
| | 0.13 | 1.325 | 1.710 | 1.360 | 1.084 | 0.891 | 0.754 | 0.651 | 0.572 | 0.510 |
| | $\mu \backslash \sigma$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
| K/S=100/105 | 0.05 | 0.039 | 0.066 | 0.055 | 0.044 | 0.037 | 0.031 | 0.027 | 0.024 | 0.021 |
| | 0.07 | 0.152 | 0.261 | 0.219 | 0.178 | 0.147 | 0.125 | 0.108 | 0.095 | 0.085 |
| | 0.09 | 0.331 | 0.583 | 0.491 | 0.399 | 0.331 | 0.282 | 0.244 | 0.215 | 0.192 |
| | 0.11 | 0.569 | 1.028 | 0.870 | 0.709 | 0.589 | 0.500 | 0.434 | 0.382 | 0.341 |
| | 0.13 | 0.861 | 1.594 | 1.355 | 1.106 | 0.919 | 0.782 | 0.678 | 0.597 | 0.533 |

Table 11: Option Returns, Stock Returns and Option-Implied Stock Returns

The first row of each panel repeats the benchmark results from Table 3. The second and third rows report the average return and volatility of stocks underlying these option portfolios. The fourth row computes expected option returns using the Black-Scholes-Merton expected option return formula given the stock data in rows 2 and 3. The last row reports the option-implied expected stock return using the data in rows 1 and 3 by inverting the Black-Scholes-Merton expected option return formula. The average option returns in the first row are monthly for consistency with Table 3. Stock returns, stock volatilities and option implied expected stock returns are annual. The sample period is from January 1996 to July 2013.

| Panel A: Call Options | | | | | |
|---|---|---|---|---|---|
| | Low | 2 | 3 | 4 | High |
| Average option return | 0.147 | 0.128 | 0.111 | 0.084 | 0.009 |
| Average stock return | 0.108 | 0.132 | 0.132 | 0.108 | 0.048 |
| Average stock volatility | 0.209 | 0.308 | 0.403 | 0.525 | 0.813 |
| Expected option return | 0.145 | 0.130 | 0.101 | 0.060 | 0.011 |
| Option-implied expected stock return | 0.106 | 0.127 | 0.139 | 0.136 | 0.041 |
| | | | | | |
| Panel B: Put Options | | | | | |
| | Low | 2 | 3 | 4 | High |
| Average option return | -0.146 | -0.153 | -0.109 | -0.077 | -0.075 |
| Average stock return | 0.108 | 0.132 | 0.132 | 0.096 | 0.048 |
| Average stock volatility | 0.214 | 0.313 | 0.408 | 0.532 | 0.827 |
| Expected Option Return | -0.124 | -0.109 | -0.083 | -0.040 | -0.005 |
| Option-implied expected stock return | 0.120 | 0.172 | 0.162 | 0.153 | 0.225 |

Table 12: Regressions of Index Option Returns on Index Volatility

Using a pooled sample of S&P 500 index options (SPX) with $0.9 \leq K/S \leq 1.1$ and one-month maturity, we report results for the regression of monthly SPX option returns on lagged index volatility:

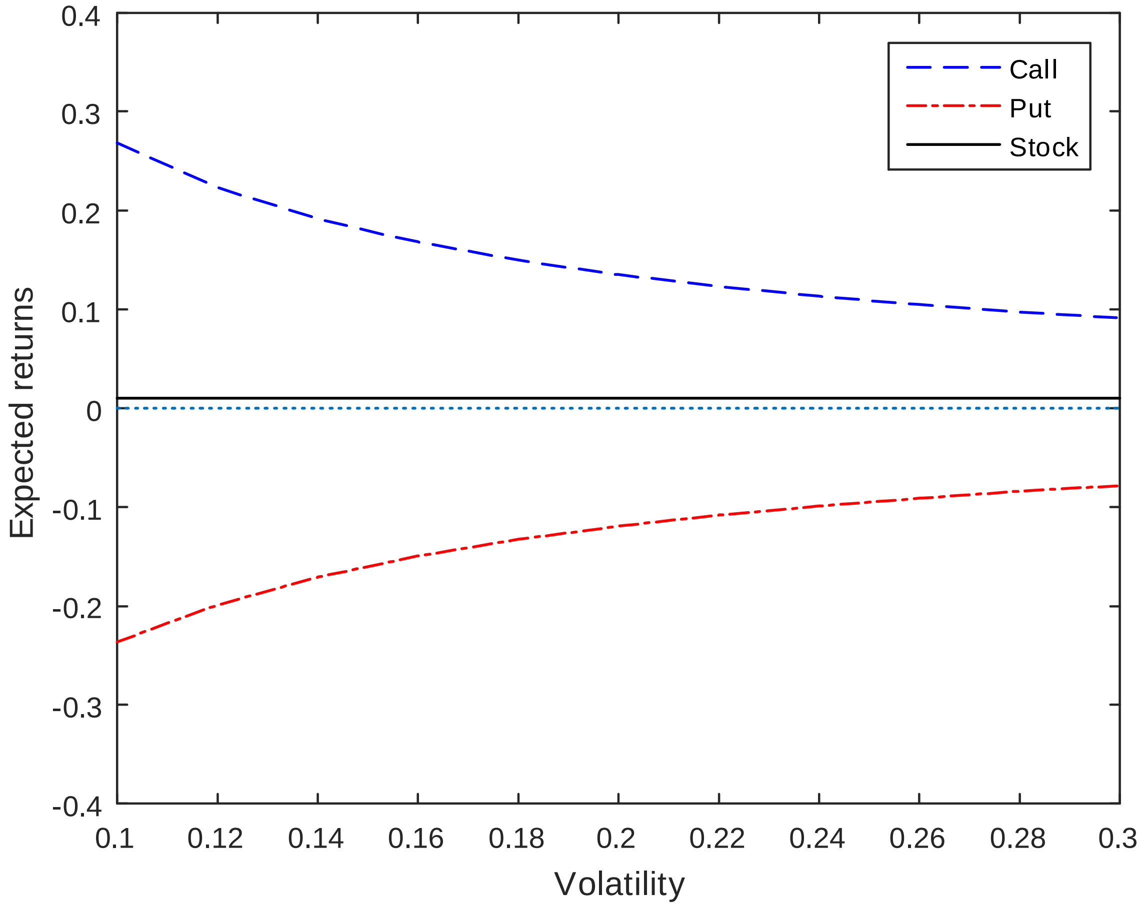
$$R_{t+1}^{i} = constant + \beta_1 VOL_t^i + \beta_2 Moneyness_t^i + \beta_3 R_t^I + \epsilon$$

where $R_{t+1}^{i}$ is the option return from month $t$ to month $t+1$, $R_t^I$ is the S&P 500 index return in month $t$ and $VOL_t$ is the index volatility. Columns (1)-(4) consider four index volatility measures: realized volatility over the previous 14 days, realized volatility over the preceding month, realized volatility over the previous 60 days, and option-implied volatility. Columns (5)-(8) consider the same regressions using only liquid SPX options, consisting of calls with $0.98 \leq K/S \leq 1.1$ and puts with $0.90 \leq K/S \leq 1.02$. The sample period is from January 1996 to July 2013. Newey-West t-statistics with four lags are reported in parentheses.

| Panel A: SPX Calls | Full sample | | | | Only liquid options | | | |
|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| Intercept | 4.091 | 4.178 | 4.285 | 7.113 | 9.273 | 9.211 | 9.128 | 10.849 |
| | (5.988) | (6.360) | (6.676) | (14.025) | (9.440) | (9.311) | (9.154) | (11.931) |
| 14 day realized vol | -0.460 | | | | -0.194 | | | |
| | (-1.913) | | | | (-0.548) | | | |
| 30 day realized vol | | -0.921 | | | | -0.858 | | |
| | | (-3.781) | | | | (-2.516) | | |
| 60 day realized vol | | | -1.456 | | | | -1.617 | |
| | | | (-4.778) | | | | (-3.767) | |
| implied vol | | | | -3.697 | | | | -4.444 |
| | | | | (-5.571) | | | | (-4.473) |
| Adjusted R-square | 1.13% | 1.22% | 1.37% | 2.01% | 1.67% | 1.73% | 1.90% | 2.54% |

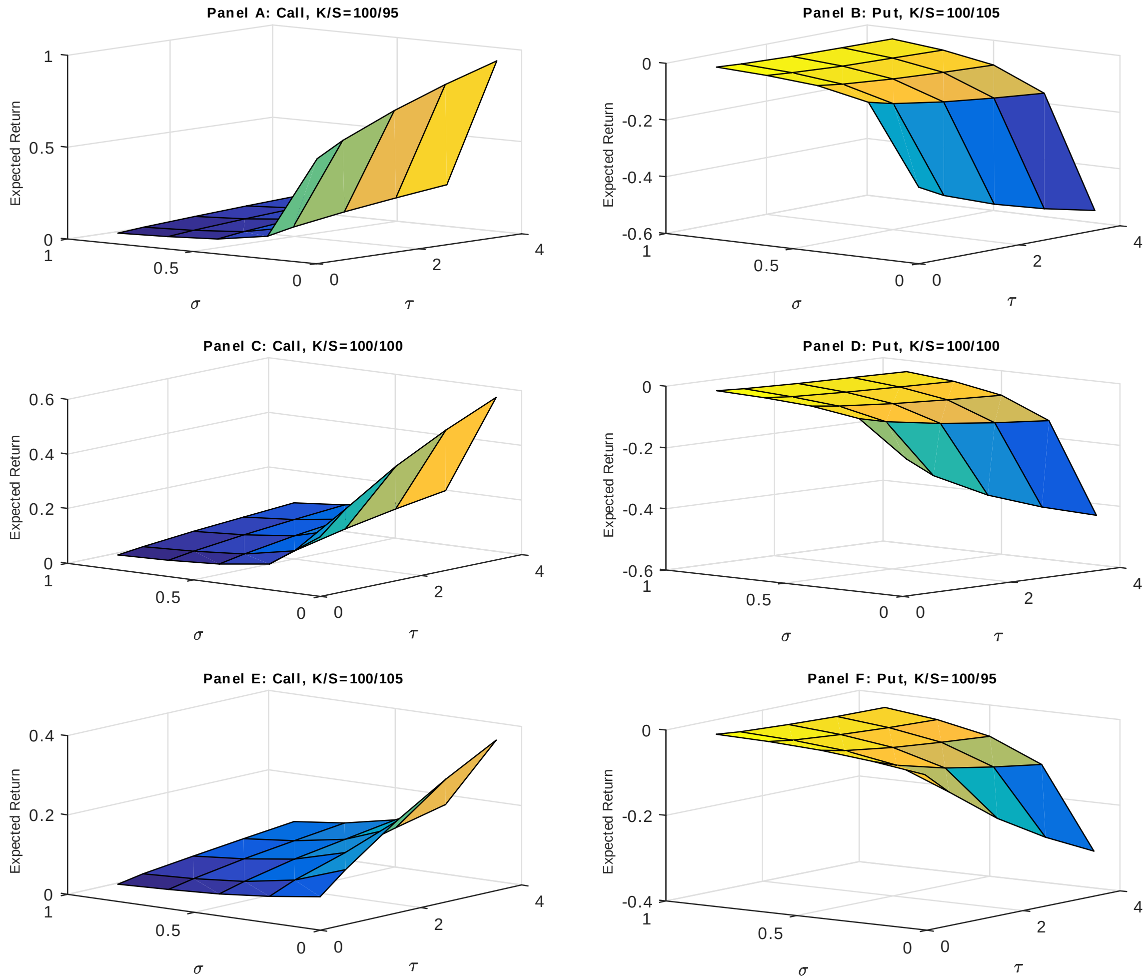
| Panel B: SPX Puts | Full sample | | | | Only liquid options | | | |
|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| Intercept | -4.243 | -4.219 | -4.216 | -4.499 | -6.324 | -6.139 | -6.069 | -6.771 |
| | (-8.418) | (-8.230) | (-8.155) | (-9.067) | (-8.971) | (-8.622) | (-8.444) | (-8.704) |
| 14 day realized vol | 2.106 | | | | 2.664 | | | |
| | (3.918) | | | | (3.881) | | | |
| 30 day realized vol | | 1.393 | | | | 1.887 | | |
| | | (2.951) | | | | (3.140) | | |
| 60-day realized vol | | | 1.070 | | | | 1.582 | |
| | | | (2.619) | | | | (2.978) | |
| implied vol | | | | 0.263 | | | | 0.920 |
| | | | | (0.652) | | | | (1.732) |
| Adjusted R-square | 2.70% | 1.73% | 1.46% | 1.13% | 3.18% | 2.06% | 1.76% | 1.27% |

Figure 1: Option Leverage as a Function of Volatility in the Black-Scholes-Merton Model



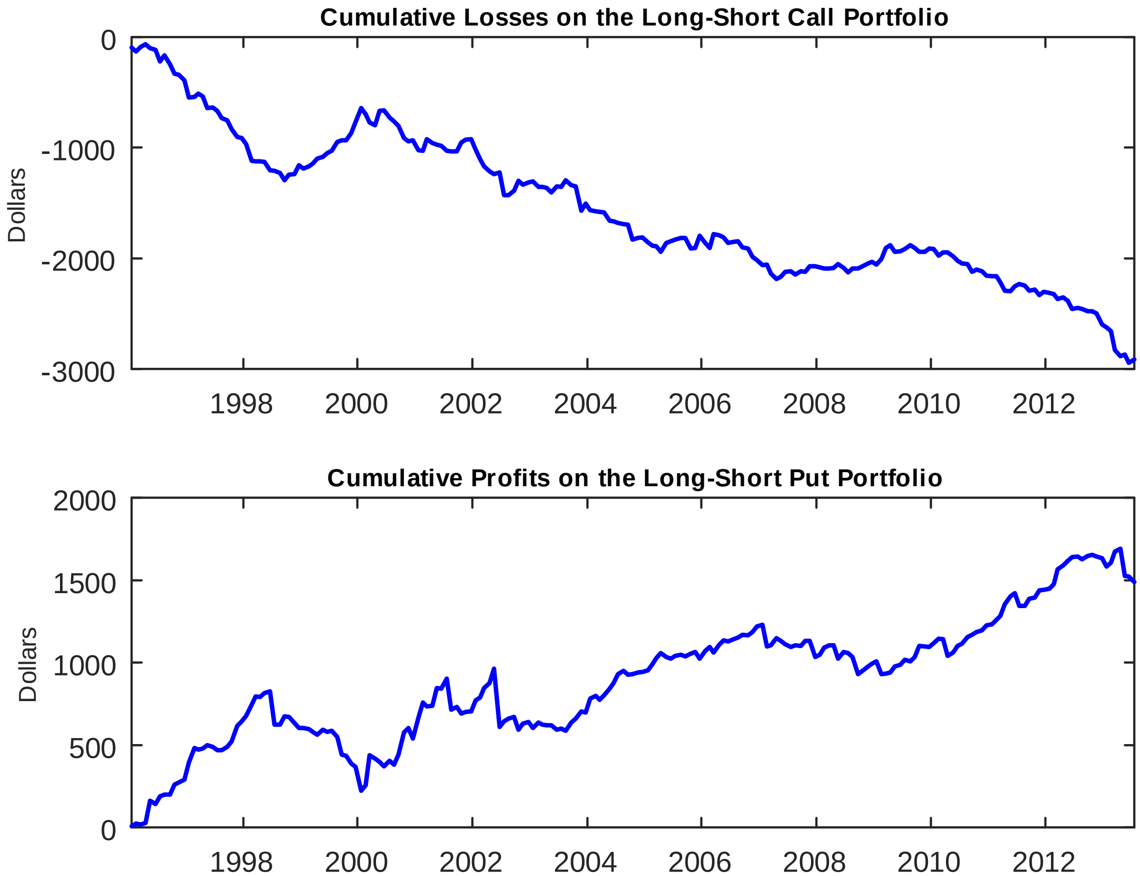

Notes: We plot expected monthly returns on a call option, a put option and the underlying stock in the Black-Scholes-Merton model. We set the expected annual return on the stock $\mu$ equal to 10% and the risk-free rate $r$ equal to 3%. Options are at-the-money (ATM) and have a maturity of 1 month.

Figure 2: Expected Option Returns in the Black-Scholes-Merton Model



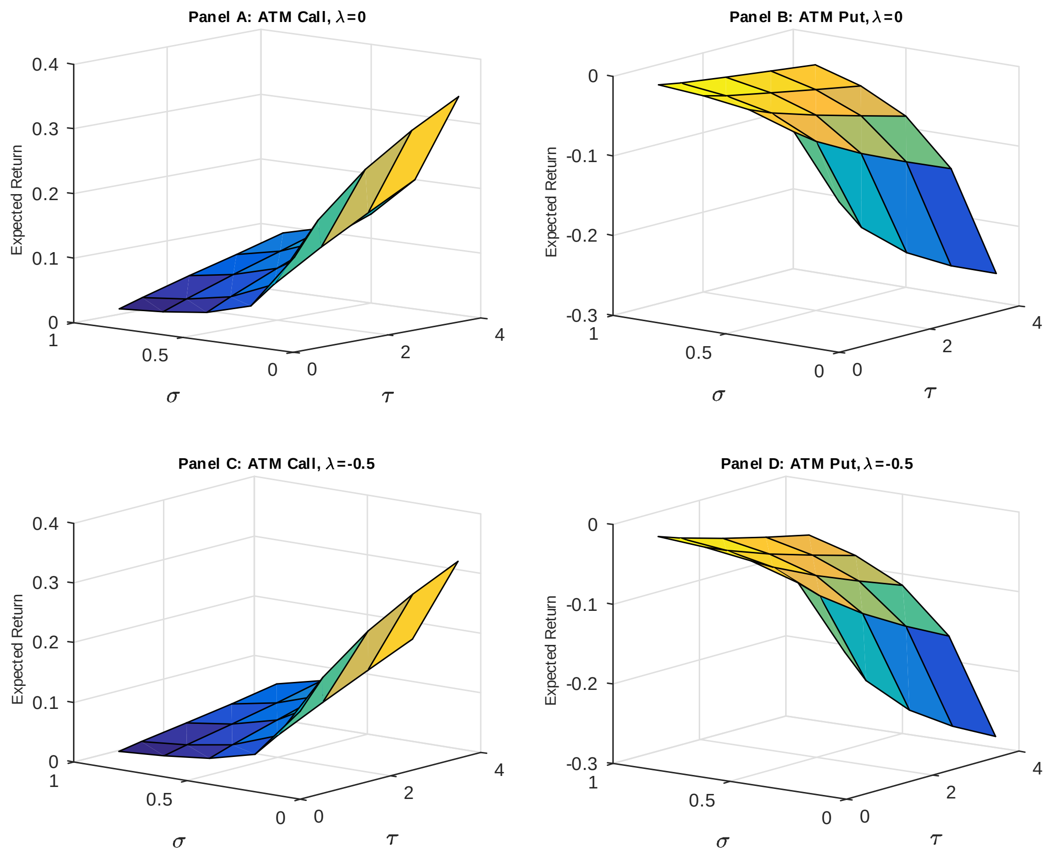

Notes: We plot expected option returns in the Black-Scholes-Merton model against volatility ($\sigma$) and time-to-maturity ($\tau$). In all computations, we set the expected return on the stock $\mu$ equal to 10% and the risk-free rate $r$ equal to 3%. We set the strike price (K) equal to 100 and the stock price (S) equal to either 95, 100 or 105. Returns are reported as raw returns for the relevant horizons.

Figure 3: Cumulative Losses and Profits on the Long-Short Portfolios




Notes: We plot the cumulative losses and profits from investing in the long-short portfolios documented in Table 3. We assume that an investor invests $100 in the long-short portfolio every month.

Figure 4: Expected Option Returns in the Heston Model




Notes: We plot expected option returns in the Heston model against volatility ($\sigma$) and time-to-maturity ($\tau$) for at-the-money (ATM) options. We use the parameter values in Panel A of Table 8, and set the expected stock return $\mu$ equal to 9.91% as in Broadie, Chernov and Johannes (2009). The volatility risk premium ($\lambda$) is either 0 (Panels A and B) or -0.5 (Panels C and D). Returns are reported as raw returns for the relevant horizons.

# Online Appendix

Table A.1: Straddle Portfolio Returns Sorted on Volatility

We report average monthly returns for five straddle portfolios sorted on the volatility of the underlying stock. We use three samples of straddles based on moneyness: $0.95 \leq K/S \leq 1$, $0.875 \leq K/S < 0.95$, and $0.80 \leq K/S < 0.875$. Every month, we select call and put options on the same stock with the same strike price and maturity to form straddles. These straddles are then sorted into five quintile portfolio based on the realized volatility over the preceding month. Portfolio Low (High) contains straddles with the lowest (highest) underlying volatility. We report equal-weighted and volume-weighed portfolio returns. Straddle volume is computed as the average volume for the call and put options that form the straddle. The sample period is from January 1996 to July 2013. Newey-West t-statistics with four lags are reported in parentheses. Statistical significance at the 10%, 5%, and 1% level is denoted by *, **, and *** respectively.

| | | | | | | |
|---|---|---|---|---|---|---|
| Panel A: $0.95 \leq K/S \leq 1$ | | | | | | |
| | Low | 2 | 3 | 4 | High | H-L |
| Equal-weighted | 0.028 | 0.010 | 0.022 | 0.014 | -0.026 | -0.054*** |
| | | | | | | (-2.904) |
| Volume-weighted | 0.026 | -0.006 | 0.005 | 0.014 | -0.037 | -0.063** |
| | | | | | | (-2.139) |
| Panel B: $0.875 \leq K/S < 0.95$ | | | | | | |
| Equal-weighted | 0.022 | 0.034 | 0.025 | 0.015 | -0.033 | -0.055*** |
| | | | | | | (-3.249) |
| Volume-weighted | 0.013 | 0.044 | -0.010 | 0.003 | -0.044 | -0.057** |
| | | | | | | (-2.522) |
| Panel C: $0.80 \leq K/S < 0.875$ | | | | | | |
| Equal-weighted | 0.020 | 0.013 | 0.014 | 0.000 | -0.065 | -0.085*** |
| | | | | | | (-4.972) |
| Volume-weighted | 0.021 | 0.001 | -0.005 | -0.021 | -0.046 | -0.067** |
| | | | | | | (-2.328) |

Table A.2: Option Portfolio Returns Using Different Weighting Methods

We report long-short monthly returns for portfolios sorted on 30-day realized volatility, using different option samples. Alternative weighting methods are used: volume weighted, open interest weighted, and option value weighted. Option value is defined as the product of option price and option open interest. The sample period is from January 1996 to July 2013. Newey-West t-statistics with four lags are reported in parentheses. Statistical significance at the 10%, 5%, and 1% level is denoted by *, **, and *** respectively.

| | Volume Weighted | Open Interest Weighted | Option Value Weighted |
|---|---|---|---|
| Panel A: Call Options | | | |
| One-month ATM | -0.182*** | -0.133*** | -0.107** |
| | (-3.557) | (-3.094) | (-2.352) |
| Two-month ATM | -0.204** | -0.235*** | -0.216*** |
| | (-2.401) | (-3.640) | (-2.893) |
| One-month ITM | -0.113*** | -0.060** | -0.066*** |
| | (-3.978) | (-2.512) | (-2.652) |
| Two-month ITM | -0.210*** | -0.188*** | -0.191*** |
| | (-3.925) | (-4.334) | (-4.456) |
| One-month OTM | -0.171** | -0.059 | -0.137* |
| | (-2.145) | (-0.897) | (-1.747) |
| Two-month OTM | -0.242** | -0.292** | -0.438*** |
| | (-1.992) | (-2.422) | (-2.985) |
| | | | |
| Panel B: Put Options | | | |
| One-month ATM | 0.073 | 0.089* | 0.052 |
| | (1.433) | (1.869) | (1.061) |
| Two-month ATM | 0.081 | 0.187*** | 0.170** |
| | (0.971) | (2.969) | (2.497) |
| One-month ITM | 0.035 | 0.099*** | 0.090*** |
| | (1.094) | (3.679) | (2.934) |
| Two-month ITM | 0.154*** | 0.134*** | 0.134*** |
| | (3.451) | (2.857) | (2.896) |
| One-month OTM | 0.268*** | 0.278*** | 0.274*** |
| | (3.448) | (4.139) | (3.481) |
| Two-month OTM | 0.310*** | 0.307*** | 0.349*** |
| | (3.047) | (2.932) | (3.706) |

Table A.3: Holding Period Option Returns Sorted on Underlying Volatility

We report one-month holding period returns of options sorted on underlying volatility. On the first trading day of each month, we collect options that expire in the following month and compute the returns of holding these options to the month end. ATM options are defined by moneyness $0.95 \leq K/S \leq 1.05$, ITM options are defined by moneyness $0.80 \leq K/S < 0.95$ for calls and $1.05 < K/S \leq 1.20$ for puts, and OTM options are defined by moneyness $1.05 < K/S \leq 1.20$ for calls and $0.80 \leq K/S < 0.95$ for puts. The sample period is from January 1996 to July 2013. Newey-West t-statistics with four lags are reported in parentheses. Statistical significance at the 10%, 5%, and 1% level is denoted by *, **, and *** respectively.

| Panel A: Call Option Portfolios | | | | | | |
|---|---|---|---|---|---|---|
| | Low | 2 | 3 | 4 | High | H-L |
| 1-month ATM | 0.010 | 0.024 | 0.008 | 0.005 | -0.047 | -0.056** |
| | | | | | | (-2.046) |
| 1-month ITM | 0.002 | 0.003 | 0.003 | -0.007 | -0.051 | -0.053*** |
| | | | | | | (-2.931) |
| 1-month OTM | 0.056 | 0.032 | 0.030 | 0.008 | -0.039 | -0.095** |
| | | | | | | (-2.482) |
| Panel B: Put Option Portfolios | | | | | | |
| | Low | 2 | 3 | 4 | High | H-L |
| 1-month ATM | -0.082 | -0.069 | -0.047 | -0.037 | -0.027 | 0.055** |
| | | | | | | (2.016) |
| 1-month ITM | -0.044 | -0.021 | -0.013 | -0.018 | 0.007 | 0.051** |
| | | | | | | (2.537) |
| 1-month OTM | -0.159 | -0.099 | -0.051 | -0.057 | -0.048 | 0.111*** |
| | | | | | | (2.915) |

Table A.4: Fama-MacBeth Regressions Using Delta-Hedged Option Returns

We report results for the Fama-MacBeth regressions $R^i_{t+1} = \gamma_{0,t} + \gamma_{1,t} VOL^i_t + \Phi_t Z^i_t + \epsilon$, where $R^i_{t+1}$ is the delta hedged option return as in Goyal and Saretto (2009), $VOL^i_t$ is the underlying stock volatility, and $Z^i_t$ is a vector of control variables that includes volatility risk premium (vrp), the stock's beta (beta), firm size (size), book-to-market (btm), momentum (mom), stock return reversal (reversal), and option characteristics such as moneyness, Delta, Vega, Gamma, and option beta. The sample consists of one-month at-the-money options. The sample period is from January 1996 to July 2013. Newey-West t-statistics with four lags are reported in parentheses. Statistical significance at the 10%, 5%, and 1% level is denoted by *, **, and *** respectively.

| | calls | | | Puts | | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Intercept | -0.001 | 0.132*** | 0.129*** | 0.003* | 0.015 | 0.017 |
| | (-0.351) | (3.604) | (3.524) | (1.906) | (0.480) | (0.555) |
| Vol | **-0.009*** ** | **-0.018*** ** | | **-0.010*** ** | **-0.018*** ** | |
| | (-3.115) | (-6.204) | | (-4.097) | (-6.124) | |
| Ivol | | | -0.021*** | | | -0.020*** |
| | | | (-5.689) | | | (-5.494) |
| Vrp | | 0.039*** | 0.038*** | | 0.033*** | 0.032*** |
| | | (11.049) | (11.036) | | (10.680) | (10.641) |
| Beta | | 0.000 | -0.001** | | 0.000 | -0.001 |
| | | (-0.054) | (-1.981) | | (0.725) | (-1.201) |
| Size | | 0.000 | 0.000 | | 0.000 | 0.000 |
| | | (-1.300) | (-1.389) | | (-1.231) | (-1.292) |
| Btm | | 0.000 | 0.000 | | 0.000 | 0.000 |
| | | (-0.088) | (-0.048) | | (0.321) | (0.334) |
| Mom | | 0.001 | 0.001 | | 0.000 | 0.000 |
| | | (0.489) | (0.507) | | (0.353) | (0.326) |
| Reversal | | -0.017*** | -0.014*** | | -0.015*** | -0.013*** |
| | | (-3.370) | (-2.888) | | (-3.458) | (-2.986) |
| Moneyness | | -0.112*** | -0.111*** | | -0.011 | -0.016 |
| | | (-3.600) | (-3.549) | | (-0.333) | (-0.484) |
| Delta | | -0.028*** | -0.027*** | | -0.005 | -0.008 |
| | | (-2.789) | (-2.707) | | (-0.617) | (-0.928) |
| Vega | | 0.000 | 0.000 | | 0.000 | 0.000 |
| | | (0.416) | (0.405) | | (0.080) | (0.042) |
| Gamma | | 0.008 | 0.008 | | 0.007 | 0.006 |
| | | (1.049) | (1.095) | | (0.722) | (0.692) |
| Option beta | | -0.000* | -0.000* | | 0.000 | 0.000 |
| | | (-1.872) | (-1.696) | | (0.900) | (0.548) |