Separating Skilled and Unskilled Fund Managers by Contract Design

Xue Dong He∗ Sang Hu† Steven Kou‡

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Abstract

Foster and Young (2010, Quarterly Journal of Economics) shows that it is very difficult to design performance fee contracts that reward skilled fund managers while screening out unskilled fund managers. In particular, none of the standard practices, such as postponing bonuses and claw back provisions, can separate the skilled and unskilled managers. We show that if (1) there is a liquidation boundary, meaning that the fund investors have the right to be informed and to liquidate their stake as soon as the fund returns is bad enough to hit the boundary, and (2) the fund manager has to use his/her own money to set up a deposit to offset the potential losses from the fund investors, then the skilled and unskilled fund managers can be separated. The deposit can be a combination of cash or an equity stake in the fund. A particular version of this type of contracts, called the first-loss scheme, is quite popular in China, and is emerging in U.S.

Key words: hedge fund, performance fee, liquidation boundary, deposit

1 Introduction

Hedge fund investors hire managers to make investments on behalf of them. Fund managers with good historical investment performance, e.g., outperforming the market index, are therefore more preferred by investors. Consider a hedge fund manager who had the following investment returns in the past years: \( m_1, m_2, \ldots, m_n \). For each year, the return is higher than the benchmark: \( m_i > R_{f,i} \),

∗Department of Industrial Engineering and Operations Research, Columbia University, S. W. Mudd Building, 500 W. 120th Street, New York, NY 10027, USA. Email: xh2140@columbia.edu.
†Risk Management Institute, National University of Singapore, 21 Heng Mui Keng Terrace, Singapore 119613. Email: rmihsa@nus.edu.sg.
‡Risk Management Institute and Department of Mathematics, National University of Singapore. Address: 21 Heng Mui Keng Terrace, 1st Building #04-03, Singapore 119613. Email: matsteve@nus.edu.sg.
The mimic strategy succeeds.  
The total return is $M = m$.

The mimic strategy fails and 
the fund goes into bankruptcy. 
The total return is $M = 0$.

Initial Capital $X_0$. 

Figure 1: Mimic Strategy in Foster and Young (2010)

e.g., $R_{f,i}$ can be the S&P 500 index return or the risk-free rate. Can these past outcomes possibly be a Madoff illusion?\footnote{See Madoff Investment Scandal at \url{http://en.wikipedia.org/wiki/Madoff_investment_scandal} for details.} Foster and Young (2010) claim it could be a mimic strategy that can be, for example, investing in the benchmark portfolio and shorting the digital options to earn the premium. By employing the mimic strategy, the fund manager can deliver the total return $m$ with probability $R_f/m$, or the total return 0 with probability $1 - R_f/m$ at each year (Foster and Young, 2010); see Figure 1 for illustration. The mimic strategy can be constructed by shorting a quantity $q$ of digital options with payoff $1_{K < S_T}$ at time $T$, and then investing the fund capital $x_0$ and the premiums collected from selling the option that equal to $qv_0 = q\mathbb{E}^*[e^{-rT}1_{K < S_T}]$, into the benchmark portfolio. The quantity $q$, strike price $K$ and maturity date $T$ are chosen in a way such that the total return equals to $m$ if options are not exercised, or the return is 0 if options are exercised. In other words, $q$, $K$, and $T$ are such that

$\begin{align*}
\frac{(qv_0 + x_0)R_f}{x_0} = m, \\
(qv_0 + x_0)R_f - q = 0, \\
\mathbb{P}(K \geq S_T) = R_f/m.
\end{align*}$

A skilled manager is one who can deliver a return $m_i$ that is higher than the benchmark $R_{f,i}$ because of his/her expertise, private information, powerful prediction, etc. An unskilled manager is one who cannot deliver a return in excess of the benchmark rate in expectation, but can mimic the skilled one by employing a mimic strategy.

The question that follows is if it is possible to separate the skilled and unskilled fund managers by contract design. If the unskilled managers employs the mimic strategy, then it is possible for
them to mimic successfully in consecutive years. The investors need to separate the skilled and unskilled fund managers, i.e., to attract the truly skillful managers and screen out the unskilled ones through contract design. Foster and Young (2010) claim that it is very difficult to separate the skilled and unskilled fund managers by designing performance fee contract. In particular, if the manager does not need to cover certain losses from investors, then the unskilled manager will enter the fund because there is some chance to gain the performance fee when the mimic strategy succeeds. Common practices such as postponing bonuses and claw back provisions cannot deter the unskilled manager, because they do not involve covering certain losses. Suppose the skilled manager is able to generate a high total return for sure due to their skills. Consider an unskilled manager mimic the skilled manager’s performance by employing the mimic strategy. If the mimic strategy succeeds, then the return is the same as the one generated by the skilled manager. If the strategy fails, then the fund goes into bankruptcy. In other words, the unskilled managers are willing to enter the fund because there is some chance to gain the performance fee, but he/she does not need to cover certain losses from investors. However, if not designed carefully, the contract does not work even requiring the manager to cover certain losses from investors, because both the skilled and unskilled managers are deterred. For example, the fund manager holds a safety stake with his/her own money putting into the risk-free asset. If investors have losses, the safety stake is used to cover certain losses. It can deter the unskilled manager, but at the same time, it also deters the skilled manager because the skilled one incurs opportunity costs by investing his/her own money into the risk-free asset while a higher investment return is available. Foster and Young (2010) conclude that it is very difficult to design performance fee contracts that reward skilled fund managers while screening out unskilled fund managers.

In this paper we show that if (1) there is a liquidation boundary (2) and the fund manager has to use his/her own money to set up a deposit, then the skilled and unskilled fund managers can be separated. A liquidation boundary is pre-determined in the contract that the fund investors have the right to be informed and to liquidate their stake once the fund returns is bad enough to hit the boundary. The fund manager uses his/her own money to set up the deposit that can be a combination of cash or equity to compensate the potential losses from fund investors.

First, the fund investors have the right to be informed once if the fund returns is bad enough to hit the liquidation boundary, and they also have the right to liquidate their stake at once. This restriction prevents the fund bankruptcy if the unskilled manager employs a mimic strategy and it fails. In other words, if the strategy succeeds, the total return rate is the same as that of the skilled manager; if it fails, the fund value hits the liquidation boundary rather than becomes zero. Second, the fund manager is required to use his/her own money to set up the deposit, which can be a combination of cash and equity and is used to cover certain losses from investors. If both the liquidation boundary and the deposit exist then the skilled and unskilled fund managers can be separated by carefully designing the performance fee contract. The contract is chosen in a way such that the unskilled manager is discouraged to enter the fund because the expected payoff of
being the fund manager is smaller than that of not, while the skilled manager is willing to enter the fund because he/she has positive gains from taking the fund manager position.

Why are the liquidation boundary and the deposit important? First, both the deposit and the liquidation boundary are necessary to separate the skilled and unskilled managers. (1) It is straightforward to see that unskilled manager does not incur losses if there is no deposit required to cover certain losses from investors potentially. Consequently, the unskilled manager is not screened out because there are still some chance to gain the performance fee if he/she implements the mimic strategy successfully. (2) If there is no liquidation boundary as discussed in Foster and Young (2010), the equity component of the deposit does not compensate any loss when its value drops to zero; the cash component of the deposit does not work either, for the reason that when the unskilled manager is screened out, the skilled one does not enter the fund either because of the opportunity cost incurred by putting his/her own money into the deposit and earning a lower rate. In summary, without deposit or liquidation boundary, the skilled and unskilled managers cannot be separated.

Second, if there exist the deposit and the liquidation boundary, the skilled and unskilled managers can be separated. A higher liquidation boundary leads to a larger probability that the mimic strategy by an unskilled manager will fail, but does not influence the skilled manager because he/she can deliver a total return that is higher than the benchmark rate. On the other hand, a non-zero liquidation boundary prevents the equity component of the deposit from dropping to zero when the mimic strategy fails. Therefore, if the deposit are fully used to cover losses for investors, the unskilled manager would suffer larger losses under a higher liquidation boundary. Hence, the unskilled manager would be screened out, while the skilled is preserved if the performance fee contract is designed carefully.

2 New Scheme

Consider a hedge fund with initial value $X_0$ at time 0. We consider the single-period setting first. Denote $M$ as the total return rate the manager of the fund can deliver in the period $[0, 1]$. Note that skilled managers can deliver higher returns than unskilled managers. Denote $R_f$ the benchmark rate. If the total return is larger than the benchmark it is considered to be gains, otherwise if the return is smaller than the benchmark it is considered as losses.

Suppose the manager holds an equity stake of the fund. Suppose the size of the stake is $wX_0$ for some $w \in [0, 1)$. Then, the investors of the fund hold $(1 - w)X_0$. Because the equity stake can also be invested, the fund asset value becomes $X_1 = X_0 M$ at the end of the period. The manager takes $wX_0 M$, which is the payoff from his own stake, and a performance fee based on the return of the investors’ stake in the fund. Suppose with initial stake $x$ and total return rate $R$ of the investors, the manager takes $\phi(x, R) = \alpha x (R - R_f)^+$ amount of performance fee, for
some $\alpha \in [0,1]$ called the incentive rate. Then, in this scheme, the manager’s performance fee is $\phi = \alpha (1 - w)X_0(M - R_f)^+$. Consequently, the total payoff of the manager is

$$wX_0M + \phi$$

This is the traditional scheme in He and Kou (2014).

Now, consider a new scheme in which the manager holds a stake with size $wX_0$ for some $w \in [0,1)$, where his/her own money is used to set up the deposit. Then, the investors hold $(1 - w)X_0$. Part of the deposit with size $\gamma wX_0$ is the cash component to be held in the benchmark portfolio, earning the return rate $R_f$, for some $\gamma \in [0,1]$, and the remaining $(1 - \gamma)wX_0$ is the equity component, earning the fund return rate. At the end of the period, the deposit is used to cover certain losses from investors first. Thus, at the end of the period, the asset value of the fund becomes

$$X_1 = \gamma wX_0R_f + (1 - \gamma)wX_0M + (1 - w)X_0M.$$ 

When the total return is higher than the benchmark, i.e., $M \geq R_f$, the payoff of the manager is the deposit plus the performance fee, i.e.,

$$\gamma wX_0R_f + (1 - \gamma)wX_0M + \phi.$$ 

When the investment return is lower than the benchmark, i.e., $M < R_f$, the fund is in a loss. In this case, the manager covers all the loss up to the deposit, so the total payoff of the manager is

$$\max(\gamma wX_0R_f + (1 - \gamma)wX_0M - (1 - w)X_0(R_f - M), 0).$$

For example, one special case of this new scheme is $\gamma = 1$, where all the deposit is of cash component, called the safety stake scheme. Another one is $\gamma = 0$, where all the deposit is of equity component. This is called the first-loss scheme discussed in He and Kou (2014). This type of contracts is quite popular in China, and is emerging in U.S..\(^2\)

### 3 Liquidation Boundary

The skilled manager can generate a total return which is strictly larger than the benchmark rate. The unskilled manager can mimic the skilled one’s performance by taking a position in the option market to bet on the outcome with the expected return equal to the benchmark rate. If the mimic strategy succeeds, then the unskilled manager performs same as the skilled one. If the

\(^2\)See a report from CBS Marketwatch on May 23, 2011 about the first-loss capital in U.S..
The mimic strategy succeeds. The total return is \( M = m \).

The mimic strategy fails and the liquidation boundary is hit. The total return is \( M = b \).

Initial Capital \( X_0 \).

The mimic strategy fails, the investors have losses. Foster and Young (2010) suppose if the mimic strategy fails, the investors lose all the initial capital. However, the investors could avoid such consequences by setting a liquidation boundary.

The liquidation boundary, \( b \in [0, 1) \), is a bottom line such that once it is hit, the investors may liquidate their stake and the fund is closed down. If the investors’ initial capital equal to \((1 - w)X_0\), then once its value falls to \(b(1 - w)X_0\), the investors should be informed right away and the fund is liquidated. If the mimic strategy fails, the fund capital value hits the liquidation boundary rather than becomes zero; see Figure 2. The investors legally stipulate in the contract that once the liquidation boundary is hit, the investors should be informed and have the right to liquidate their stake. This restriction prevents the unskilled manager from employing a mimic strategy which will cause all the capital lost if it fails. With the existence of the liquidation boundary the investors are guaranteed to remain at least \( b \) percentage of their initial stake at the end of the period.

Suppose the skilled manager can generate a total return equal to \( m \), which is strictly larger than the benchmark rate, i.e., \( m > R_f \).

**Definition 1** A strategy is called a mimic strategy if the total return \( M = m \) with probability \( p_s \), or \( M = b \) with probability \( 1 - p_s \), and \( E[M] = R_f \).

We have \( p_s = \frac{R_f - b}{m - b} \), \( 1 - p_s = \frac{m - R_f}{m - b} \). Moreover, the probability that the mimic strategy succeeds, \( p_s \), is smaller for higher liquidation boundary \( b \). On the contrary, the failure probability, \( 1 - p_s \), is larger for higher liquidation boundary \( b \). It is straightforward to observe that the unskilled manager does not add value to the investors because the expected return is just the benchmark
rate, while the investors can obtain such a return with smaller variance by investing into the benchmark portfolio directly.

The mimic strategy, for example, can be constructed by shorting barrier options and investing the fund capital and the option premiums into the benchmark portfolio. Denote $q$ the quantity of barrier options, $x_0$ the fund capital, and $v_t$ is the option price at time $t$. Therefore, $qv_0$ is the total premiums from selling the options. The barrier is triggered when its option price exceeds a threshold; otherwise, it degenerates into a digital option with strike price $K$ at the maturity date $T$. Denote $\tau$ the option knock-out time,

$$\tau := \inf\{t \geq 0 | v_t \geq v_0 R_{f,t} + \frac{R_{f,t} - b}{m - b}\}.$$ 

To be more specific, the payoff of the option is

$$\begin{cases} 
1_{K < S_T}, & \text{at time } T \text{ if } \tau > T, \\
v_\tau, & \text{at time } \tau \text{ if } \tau \leq T.
\end{cases}$$

In other word, if the knock-out is triggered before $T$, i.e., $\tau \leq T$, the option is redeemed by the seller; otherwise its payoff is same as a digital option. The quantity $q$, strike price $K$, and maturity date $T$ are such that

$$\begin{cases} 
\frac{(qv_0 + x_0) R_f}{x_0} = m, \\
\frac{(qv_0 + x_0) R_f - q}{x_0} = b, \\
\mathbb{P}(K \geq S_T, \tau > T) = p_s.
\end{cases}$$

4 Contract Design

In this section we show that the skilled and unskilled managers can be separated by contract design. In particular, if there is a liquidation boundary and fund managers are requested to set up a deposit to cover the loss, the unskilled managers are discouraged to enter the fund while the skilled are still kept, for the incentive rate of the performance fee contract to be chosen properly.

**Theorem 1** (i) If there is no deposit, i.e., $w = 0$, the skilled and unskilled managers cannot be separated.

(ii) If there is no liquidation boundary, i.e., $b = 0$, the skilled and unskilled managers cannot be separated.

3\(^\)See the discussion of dual purpose funds in Ingersoll (1976) and Dai et al. (2015).
(iii) Suppose $w \in (0, 1)$ and $b \in (0, 1)$. If

$$\frac{\gamma w}{1 - w} < \alpha < \frac{\gamma w R_f + (1 - \gamma) w b}{(1 - w)(R_f - b)},$$

then the skilled and unskilled managers can be separated.

**Proof** (i) If the fund manager is not required to use his/her own money to set up the deposit, the unskilled manager then has a positive expected gain of the performance fee, equal to $\phi p_s > 0$. In particular, under the traditional scheme where the manager holds an equity stake of the fund but is not required to cover certain losses from investors, then the unskilled one is not discouraged. If the mimic strategy succeeds, the payoff of the unskilled manager is $w X_0 m + \phi$. If the mimic fails, his payoff is $w b X_0$. Consequently, the expected payoff is

$$(w X_0 m + \phi) p_s + w b X_0 (1 - p_s) = w R_f X_0 + \phi p_s > w R_f X_0,$$

meaning that unskilled manager cannot be discouraged under the traditional scheme where there is no covered losses for investors.

(ii) Suppose there is no liquidation boundary. Thus, for the unskilled manager, the total return is $m$ with probability $p_s = \frac{R_f}{m}$ if the mimic strategy succeeds, or the total return is zero with probability $1 - p_s = 1 - \frac{R_f}{m}$ if the mimic strategy fails. Suppose for the manager holding a stake with size $w x$ and the total return $R$, the payoff on his/her stake is $\psi(w x, R)$. If the mimic strategy succeeds, the total payoff of the unskilled manager is $\psi(w X_0, m) + \phi((1 - w) X_0, m)$. If the mimic strategy fails, the total payoff is $\psi(w X_0, 0) + \phi((1 - w) X_0, 0)$. If the unskilled one does not enter the fund, his/her payoff is just equal to $w R_f X_0$. To rule out the unskilled manager, $\psi$, $\phi$ should be such that the expected payoff for being the fund manager is less than that of not, i.e.,

$$w R_f X_0 > (\psi(w X_0, m) + \phi((1 - w) X_0, m)) p_s + (\psi(w X_0, 0) + \phi((1 - w) X_0, 0))(1 - p_s)$$

$$\geq (\psi(w X_0, m) + \phi((1 - w) X_0, m)) \frac{R_f}{m}.$$

On the other hand, the skilled manager can deliver the return $m$ and hence he/she will gain the performance fee $\phi$. If the skilled one do not enter the fund, his/her payoff would be $w X_0 m$. To attract the skilled manager, $\psi$, $\phi$ should be such that the payoff for being the fund manager is larger than that of not, i.e.,

$$\psi(w X_0, m) + \phi((1 - w) X_0, m) > w X_0 m.$$

It is then straightforward to observe that above two inequality cannot hold simultaneously. Hence, the skilled and unskilled managers are not separated if the liquidation boundary $b = 0$. 

8
(iii) Recall that if the mimic strategy succeeds, the payoff for the unskilled manager is 
$\gamma wX_0 R_f + (1 - \gamma) wX_0 m + \phi$, and if the mimic fails, his/her payoff is 
$max(\gamma wX_0 R_f + (1 - \gamma) wX_0 b - (1 - w) X_0 (R_f - b), 0)$. As a result, the expected payoff of the unskilled manager is

$$(\gamma wX_0 R_f + (1 - \gamma) wX_0 m + \phi) p_s + \max(\gamma wX_0 R_f + (1 - \gamma) wX_0 b - (1 - w) X_0 (R_f - b), 0)(1 - p_s)$$

$$= (\gamma wX_0 R_f + (1 - \gamma) wX_0 m) p_s + (\gamma wX_0 R_f + (1 - \gamma) wX_0 b)(1 - p_s)$$

$$+ \phi p_s - \min(\gamma wX_0 R_f + (1 - \gamma) wX_0 b, (1 - w) X_0 (R_f - b))(1 - p_s)$$

$$= \gamma wX_0 R_f + (1 - \gamma) wX_0 R_f + \alpha (1 - w) X_0 (m - R_f)p_s$$

$$- \min(\gamma wX_0 R_f + (1 - \gamma) wX_0 b, (1 - w) X_0 (R_f - b))(1 - p_s)$$

$$= wX_0 R_f + \alpha (1 - w) X_0 (m - R_f) \frac{R_f - b}{m - b} - \min(\gamma wX_0 R_f + (1 - \gamma) wX_0 b, (1 - w) X_0 (R_f - b)) \frac{m - R_f}{m - b}.$$ 

If the unskilled manager does not enter the fund, his/her payoff would be $wX_0 R_f$. If the expected payoff of being the fund manager is smaller than that of not, then the unskilled manager is screened out, i.e,

$$wX_0 R_f > wX_0 R_f + \alpha (1 - w) X_0 (m - R_f) \frac{R_f - b}{m - b}$$

$$- \min(\gamma wX_0 R_f + (1 - \gamma) wX_0 b, (1 - w) X_0 (R_f - b)) \frac{m - R_f}{m - b},$$

which gives the upper bound of the incentive rate $\alpha$. The incentive rate $\alpha$ is such that

$$\alpha < \frac{\gamma w R_f + (1 - \gamma) wb}{(1 - w)(R_f - b)}.$$

On the other hand, the skilled manager is willing to enter the fund if the expected payoff of being the fund manager is larger than that of not. In other words, the opportunity cost that the skilled manager is required to put his/her own money into the deposit must be compensated by the performance fee. If the skilled one enters the fund, his/her payoff is $\gamma wX_0 R_f + (1 - \gamma) wX_0 m + \phi$; otherwise, the payoff is $wX_0 m$.

$$wX_0 m < \gamma wX_0 R_f + (1 - \gamma) wX_0 m + \phi = \gamma wX_0 R_f + (1 - \gamma) wX_0 m + \alpha (1 - w) X_0 (m - R_f),$$

which gives the lower bound of $\alpha$, that is,

$$\alpha > \frac{\gamma w}{1 - w}.$$ 

Q.E.D

If there is no deposit from his/her own money, the manager does not need to cover certain losses from investors. If there is no liquidation boundary, the equity component of the deposit
cannot cover any loss from investors when its value drops to zero, and the cash component of the deposit does not work because it deters both the skilled and unskilled managers.

Why do we have the upper bound and lower bound of $\alpha$? For the unskilled manager, the expected loss from the deposit must be larger than the expected gain from the performance fee so $\alpha$ has an upper bound. For the skilled manager, the opportunity cost of investing his/her own money into the risk-free asset must be compensated by the performance fee so $\alpha$ has a lower bound. The unskilled manager would require a higher incentive rate for a higher liquidation boundary, while the skilled one is not influenced.

**Proposition 2** When the deposit is fully used to cover the potential loss, unskilled managers’ expected payoff decreases as the liquidation boundary $b$ increases.

**Proof** If the amount of the deposit is larger than that of the loss to be covered when the mimic fails, i.e., $\gamma wX_0R_f + (1-\gamma)wX_0b \geq (1-w)X_0(R_f - b)$, then unskilled managers have an expected loss for any $\alpha \in [0, 1]$, which equal to

$$\alpha(1-w)X_0(m-R_f)p_s - (1-w)X_0(R_f - b)p_f$$

$$= \alpha(1-w)X_0(m-R_f)\frac{R_f - b}{m-b} - (1-w)X_0(R_f - b)\frac{m-R_f}{m-b}$$

$$= -(1-\alpha)X_0\frac{(m-R_f)(R_f - b)}{m-b} < 0.$$ 

In other words, if $b$, $w$, $\gamma$ and $R_f$ are such that the amount of the deposit is larger than that of the potential loss to be covered, unskilled managers will not enter the fund. Hence, we only need to consider the case when the deposit are fully used to compensate the loss, i.e., $\gamma wX_0R_f + (1-\gamma)wX_0b < (1-w)X_0(R_f - b)$. Then unskilled managers have an expected gain/loss equal to

$$\alpha(1-w)X_0(m-R_f)p_s - (\gamma wX_0R_f + (1-\gamma)wX_0b)p_f$$

$$= \alpha(1-w)X_0(m-R_f)\frac{R_f - b}{m-b} - (\gamma wX_0R_f + (1-\gamma)wX_0b)\frac{m-R_f}{m-b}.$$ 

Observe that the above value decreases when $b$ increases. In particular, if unskilled managers have an expected gain for some $\alpha$, then the amount of this expected gain decreases as $b$ increases.

Q.E.D

The unskilled managers’ expected gain decreases as $\alpha$ decreases. Hence, the incentive rate $\alpha$ can not be too high so as to discourage unskilled managers. Moreover, when $\gamma = 1$ corresponding to the safety stake scheme, the upper bound of $\alpha$ is larger than its counterpart in the first-loss scheme when $\gamma = 0$. The reason is because in the first-loss scheme when the mimic strategy fails, the unskilled managers’ own equity stake also suffers a loss and what remain are used to
compensate the investors’ loss, while the managers’ own safety stake does not suffer a loss when the mimic strategy fails. As a result, the expected amount used to cover the investors’ loss in the first-loss scheme are less than the expected amount in the safety stake scheme. In other words, the first-loss scheme is more attractive to the unskilled managers than the safety stake scheme is.

On the other hand, the fund need to attract the skilled managers so incentive rate $\alpha$ can not be too low. The lower bound of $\alpha$ in the safety stake scheme is also larger than its counterpart in the first-loss scheme, which is zero. The reason is because the skilled managers suffer opportunity costs in the safety stake scheme when their own can not be invested into the equity where a higher return can be earned for sure, while no opportunity cost occurs in the first-loss scheme. Hence, the first-loss scheme is also more attractive to skilled managers than the safety stake scheme is.

We take a look at some numerical examples. Let $r_{f,1} = 5\%$. 

Figure 3: Range of the incentive rate $\alpha$ as a function of the liquidation boundary $b$. Percentage of the fund managers’ deposit $w = 10\%$. $r_{f,1} = 5\%$. Percentage of cash of the deposit $\gamma = 0$ (left), $\gamma = 0.5$ (middle), $\gamma = 1$ (right). For example, if $b = 70\%$, the range of the incentive rate $\alpha$ is (0, 22.2), (5.56, 27.8), and (11.1, 33.3) with respect to $\gamma = 0$, $\gamma = 0.5$, and $\gamma = 1$ respectively.

Figure 4: Range of the incentive rate $\alpha$ as a function of the liquidation boundary $b$. Percentage of the fund managers’ deposit $w = 15\%$. $r_{f,1} = 5\%$. Percentage of cash of the deposit $\gamma = 0$ (left), $\gamma = 0.5$ (middle), $\gamma = 1$ (right). For example, if $b = 70\%$, the range of the incentive rate $\alpha$ is (0, 35.3), (8.82, 44.1), and (17.6, 52.9) with respect to $\gamma = 0$, $\gamma = 0.5$, and $\gamma = 1$ respectively.
5 Extensions

5.1 Multiple Periods

Consider $T$ periods of fund investment horizon. Denote $X_{t-1}$ the value of fund at the beginning of period $[t-1, t]$. $r_{f,t}$ is the risk free return rate in the period $[t-1, t]$, and $R_{f,t} = 1 + r_{f,t}$. $b_t$ is the liquidation boundary of period $[t-1, t]$. Suppose the managers hold a constant percentage of the fund at each period, i.e., the managers’ stake is $wX_{t-1}$ at the beginning of period $[t-1, t]$ which is used to set up the deposit to cover the potential loss of investors, among which cash component has a size of $\gamma wX_{t-1}$ and equity $(1-\gamma)wX_{t-1}$. Once the investors’ stake value falls to $b_t(1-w)X_{t-1}$ during the period $[t-1, t]$, the investors liquidate their stake immediately and the fund is closed down.

Suppose the skilled managers are able to deliver a return rate $m_t$ higher than the risk-free rate in the period $[t-1, t]$ for sure, i.e., $m_t > R_{f,t}$. On the other hand, the unskilled managers have a mimic strategy such that either the mimic succeeds with probability $p_t^s$ and the return rate is $m_t$, which is same as the skilled managers do, or the mimic fails with probability $p_t^f$ and the return rate is $b_t$, leading to the liquidation of the fund. $p_t^s + p_t^f = 1$ are such that the expected return of the unskilled managers equal to the risk-free return $R_{f,t}$. It is then straightforward to derive $p_t^s = \frac{R_{f,t} - b_t}{m_t - b_t}$ and $p_t^f = \frac{m_t - R_{f,t}}{m_t - b_t}$.

Let $d_t = \frac{1}{\prod_{n=1}^{t-1} R_{f,n}^{w X_{t-1} R_{f,n} - X_{t-1}} b_n}$ to be the discount factor. $L_t = \min(\gamma w X_{t-1} R_{f,t} + (1-\gamma)wX_{t-1}(R_{f,t} - b_t))$ is the potential covered losses. $G_t = (1-w)X_{t-1}(m_t - R_{f,t})$ is the possible gains at time $t$.

**Theorem 3** Suppose $w \in (0, 1)$, $b \in (0, 1)$. If the incentive rate $\alpha$ is such that

$$\frac{\gamma w}{1-w} < \alpha < \frac{\sum_{t=1}^{T} \left( d_t L_t p_t^f \right)}{\sum_{t=1}^{T} \left( d_t G_t p_t^s \right)},$$

then the skilled and unskilled managers can be separated.

**Proof** If unskilled managers’ mimic strategies succeed during the first $t-1$ periods, the return rates are $m_1, m_2, ..., m_{t-1}$ for the first $t-1$ periods. Note the probability that the mimic continues to succeed in the first $t-1$ period is $\prod_{n=1}^{t-1} p_n^s$. By previous analysis, the expected gain or loss for
the unskilled manager during the period \([t - 1, t]\) is

\[
V_t = \phi((1 - w)X_{t-1}, m_t)p_s^t - \min(\gamma w X_{t-1} R_{f,t} + (1 - \gamma)w X_{t-1} b_t, (1 - w) X_{t-1} (R_{f,t} - b_t))p_f^t
\]

\[
= \alpha(1 - w)X_{t-1}(m_t - R_{f,t})\frac{R_{f,t} - b_t}{m_t - b_t}
- \min(\gamma w X_{t-1} R_{f,t} + (1 - \gamma)w X_{t-1} b_t, (1 - w) X_{t-1} (R_{f,t} - b_t))\frac{m_t - R_{f,t}}{m_t - b_t}
= \alpha G_t p_s^t - L_t p_f^t.
\]

The discounted value of expected gain or loss during period \([t - 1, t]\) is

\[
\sum_{t=1}^{T} \left[ \frac{1}{\prod_{n=1}^{t-1} R_{f,n}} \left( \prod_{n=1}^{t-1} p_n^s \right) V_t \right] < 0.
\]

Hence, the unskilled managers are screened out if they have expected losses in total, i.e.,

\[
\sum_{t=1}^{T} \left[ \frac{1}{\prod_{n=1}^{t-1} R_{f,n}} \left( \prod_{n=1}^{t-1} p_n^s \right) V_t \right] < 0.
\]

For the skilled managers, the opportunity cost of putting their own money into the risk-free account during the period \([t - 1, t]\) is \(\gamma w X_{t-1}(m_t - R_{f,t})\). On the other hand, they can earn the
performance fee $\phi((1 - w)X_{t-1}, m_t)$. The discounted gain or loss of period $[t - 1, t]$ is

$$\frac{1}{\prod_{n=1}^t R_{f,n}} (\phi((1 - w)X_{t-1}, m_t) - \gamma w X_{t-1}(m_t - R_{f,t}))$$

$$= \frac{1}{\prod_{n=1}^t R_{f,n}} (\alpha(1 - w)X_{t-1}(m_t - R_{f,t}) - \gamma w X_{t-1}(m_t - R_{f,t}))$$

$$= \frac{1}{\prod_{n=1}^t R_{f,n}} (\alpha(1 - w) - \gamma w)X_{t-1}(m_t - R_{f,t}).$$

Hence, the skilled managers are willing to enter the fund if they have gains in total, i.e.,

$$\sum_{t=1}^T \left[ \frac{1}{\prod_{n=1}^t R_{f,n}} (\alpha(1 - w) - \gamma w)X_{t-1}(m_t - R_{f,t}) \right] > 0,$$

which gives the lower bound $\alpha > \frac{\gamma w}{1-w}$.

**Q.E.D**

### 5.2 Postponing Bonuses and Clawback Provisions

Other practice such as postponing bonuses and clawback provisions cannot separate the skilled and unskilled managers if they do not hold stakes of the fund and are not required to compensate the fund with their own money when the fund investment fails.

To be more specific, consider the measure of postponing bonuses first. Suppose at time $t$ the manager’s bonus is $\phi\{\{X_n, m_n\}_{n \leq t}\} \geq 0$ which is based on the historical performance, but is then postponed to be paid after $u$ periods. The present value of this postponed bonus is

$$\frac{\phi\{\{X_n, m_n\}_{n \leq t}\}}{\prod_{n=1}^{t+u} R_{f,n}},$$

and unskilled managers can earn such amount if the mimic strategy workable at least until time $t + u$ with the probability

$$\prod_{n=1}^{t+u} \frac{R_{f,n} - \bar{b}_n}{m_n - \bar{b}_n}.$$

The total discounted expected payoff of unskilled managers is

$$\sum_{t=1}^T \left\{ \frac{\phi\{\{X_n, m_n\}_{n \leq t}\}}{\prod_{n=1}^{t+u} R_{f,n}} \prod_{n=1}^{t+u} \frac{R_{f,n} - \bar{b}_n}{m_n - \bar{b}_n} \right\}.$$

Although this measure of postponing bonuses may decrease unskilled managers’ incentive to enter
the fund, it cannot get rid of them because no penalty is imposed for losing the money and the expected payoff for unskilled managers entering the fund is positive.

On the other hand, clawback provision requires the manager to give back bonuses that are paid to them in previous “good years” to compensate the fund when they lose. If the mimic strategy succeeds until time $T$, the bonus is $\phi(\{X_n, m_n\}_{n \leq T})$. If the mimic succeeds for the first $t - 1$ periods and fails at time $t$, the payoff is

$$\max \left\{ \sum_{n=1}^{t-1} \left\{ \phi(\{X_j, m_j\}_{j \leq n}) \prod_{j=n+1}^{t} R_{f,j} \right\} - X_{t}(R_{f,t} - \bar{b}_t), 0 \right\}.$$  

The total discounted expected payoff of unskilled managers is

$$\phi(\{X_n, m_n\}_{n \leq T}) \frac{\prod_{n=1}^{T} R_{f,n} - \bar{b}_n}{m_n - \bar{b}_n} \prod_{n=1}^{T} R_{f,n} - \bar{b}_n \phi(\{X_j, m_j\}_{j \leq n}) \prod_{j=n+1}^{t} R_{f,j} - X_{t}(R_{f,t} - \bar{b}_t), 0 \right\} \prod_{n=1}^{T} R_{f,n} - \bar{b}_n \prod_{n=1}^{t} R_{f,n} - \bar{b}_n \prod_{n=1}^{t} R_{f,n} - \bar{b}_n.$$  

Hence, unskilled managers are not discouraged with this positive expected payoff. As a result, clawback provisions cannot separate the skilled and unskilled managers.

References

