A Rational Explanation of Disposition Effect: Portfolio Rebalancing with Transaction Costs

Min Dai
Department of Mathematics and Risk Management Institute, NUS

Hong Liu
Olin Business School, Washington University in St. Louis and CAFR

Jing Xu
Department of Mathematics and Risk Management Institute, NUS

This Version: May 7, 2015

*We thank Dimitri Vayanos and Kenneth Singleton for helpful comments. Authors can be reached at matdm@nus.edu.sg, liuh@wustl.edu, and matxuji@nus.edu.sg.
A Rational Explanation of Disposition Effect: Portfolio Rebalancing with Transaction Costs

Abstract

Disposition effect has been widely documented and behavioral types of explanations have dominated in the literature. In this paper, we develop an optimal portfolio rebalancing model in the presence of transaction costs and committed consumption. We show that almost all of the disposition effect patterns found in the existing literature are consistent with optimal trading strategies implied by our model. In addition, selling winners that subsequently outperform held losers on average can be optimal. Therefore, it becomes an empirical question how much disposition effect cannot be explained by optimal portfolio rebalancing.

*Journal of Economic Literature* Classification Numbers: G11, H24, K34, D91.
Keywords: Disposition Effect, Portfolio Selection, Transaction Costs, Wealth Constraints
1 Introduction

Disposition effect, i.e., investors tend to realizes gains more often than losses, has been widely documented in the empirical literature. For example, using data containing 10,000 stock investment accounts in a U.S. discount brokerage from 1987 through 1993, Odean (1998) conducts a careful set of tests of the disposition effect hypothesis and concludes that disposition effects exists across years and investors.\(^1\) Behavioral types of explanations such as loss aversion, mental accounting, regret aversion have dominated in the literature.\(^2\) Lakonishok and Smidt (1986) found some empirical evidence that portfolio rebalancing may account for some of the observed trading behavior. However, Odean (1998) concludes that the disposition effect does not appear to be motivated by portfolio rebalancing and investors sell winners that tend to outperform losers they keep subsequently. In this paper, we develop an optimal portfolio rebalancing model in the presence of transaction costs and committed consumption to show that almost all of the disposition effect patterns found in the existing literature are consistent with the optimal trading strategies implied by our model. In addition, our model can also help explain why investors may sell winners that subsequently outperform losers they hold.

More specifically, we consider a model where an investor can trade in a risk free asset and multiple stocks to maximize the expected utility from the final wealth at the terminal date. Trading in any of the stocks are subject to fixed transaction costs. We then solve for the optimal trading strategies and use Monte Carlo simulations to compute the disposition measures defined by Odean (1998). We show that our portfolio rebalancing model can not only generate the disposition effect but also match the magnitude found in the empirical literature. For example, for some reasonable parameter values with 10 stocks in a portfolio, the ratio of realized gains to the sum of realized gains and paper gains (PGR) is about 0.131, while the ratio of realized losses to the sum of realized losses and paper losses (PLR) is about 0.035. For comparison, Odean (1998) reports the ratios of 0.148 and 0.098 respectively (Table I). In addition, among all the sales, gains account for 88.8%. This suggests that an investor is much more likely to realize a gain than a loss. The main intuition for why our model can generate disposition effect is as follows. In the absence of transaction costs, it is optimal for the investor to keep a certain optimal exposure in a stock, which implies that when the stock price rises (which results in a higher exposure than the optimal) the investor sells and when the stock price drops (which results in a lower exposure than the optimal) the investor

\(^1\) See also, Shefrin and Statman (1985), Grinblatt and Kelohar (2001), Kumar (2009), Ivkovi and Weisbenner (2009).

\(^2\) See for example, Shefrin and Statman (1985) and Odean (1998).
buys. This clearly implies that the investor realizes gains more often than losses. However, if there are no transaction costs, the investor trades continuously and the resulting optimal trading strategy cannot match the empirical findings. With the fixed transaction costs, as in our model, only when the value of the stock position exceeds a certain threshold (sell boundary) the investor sells, and only only when the value of the stock position drops below a lower threshold (buy boundary) the investor buys. Therefore, the investor optimally trade infrequently and every time the investor trades, a lump sum transaction is made, which is consistent with empirical evidence. On the other hand, the presence of transaction costs does not change the qualitative conclusion that to keep the risk exposure within a range, the investor realizes gains more often than losses. This is because when the sell boundary is reached, it is more likely that stock price has increased and thus it is more likely that the investor has a gain at a sale. In contrast, after a decrease in the stock price, it is more likely that the investor has a loss, but the sell boundary is less likely to be reached and thus the investor either does not trade or buy some additional amount (if the buy boundary is reached). This force of keeping risk exposure in a certain range generates the trading pattern that is consistent with the disposition effect. By varying the number of stocks, we can also match the magnitude of the disposition effect found in the literature. Intuitively, when there is only one stock, then both PGR and PLR are equal to one because conditional on a sale in the stock, there are no paper gains or paper losses at the sale time due to the absence of other stocks. As the number of stocks increases the number of stocks with paper gains increases because more stocks are in the no-transaction regions and thus both PGR and PLR tend to decrease.

Our model can also help explain why investors may sell winners that subsequently outperform losers they hold. The intuition is that if the expected return of a stock increases with a predictive state variable that is positively correlated to the stock return, then conditional on a positive shock to the stock return, the average level of the predictive state variable is greater than that conditional on a negative return. Therefore, the expected return of a winner can subsequently outperform the expected return of a loser. However, to achieve optimal risk exposure, it can still be optimal to sell winners and to hold losers due to transaction costs.

For a portfolio rebalancing model with multiple stocks but without transaction costs (e.g., Merton (1972)), it is optimal to buy some amount of another stock after a sale of a stock to rebalance the risk exposure. Odean (1998) finds that even among the sales after which there are no new purchases in three weeks, the disposition effect still exists and thus suggests that portfolio rebalancing can unlikely explain the disposition effect in this subsample. To see if our model can also generate disposition effect conditional on there is no purchase of
another stock after a sale, we restrict to the sample paths along which there is no additional purchase of another stock in three weeks after a sale. We find that our model can also produce disposition effect in this subsample with a PGR of 0.120 and a PLR of 0.03. Intuitively, in the presence of transaction costs, when it is optimal to sell a stock, it is possible (and likely) that for other stocks the risk exposures are still inside the respective optimal ranges and thus it can be optimal not to buy any of other stocks after a sale.

Odean (1998) also considers a subsample where investors sell the entire position of a stock and shows that even in this subsample, the disposition effect still appears. He suggests that portfolio rebalancing motives can unlikely explain the disposition effect in this subsample, because for portfolio rebalancing purposes, an investor is unlikely to sell the entire position because keeping a positive exposure to the stock risk is optimal. We propose an extension of our main model to include committed consumption that is widely documented for a large portion of households (e.g., Fratantoni (2001), Chetty and Szeidl (2007)) to show that it can be optimal to liquidate the entire position of a stock for portfolio rebalancing purposes and in addition, disposition effect arises even when the investor sells all the holdings in a stock. With a committed consumption level of \( C \), an investor must invest at least \( C \) in the risk free asset, if the interest rate is zero. As shown in Liu (2014), with committed consumption and leverage/short-selling constraints, it can be optimal for an investor to sell the entire position in a stock when wealth is low. Intuitively, when the wealth level is just above \( C \), the investor only invests in the stock with the highest expected return, because when the amount that can be invested in risky assets is small enough, expected return dominates risk concerns in choosing which stock to buy. Therefore, if the investor initially has positions in two stocks, when wealth drops low enough, the investor optimally sells the entire position in the stock with a lower expected return. We show that indeed in the subsample paths along which the investors sells the entire position of a stock, disposition effect can arise from following the optimal trading strategies. This typically occurs when the two stocks have a negative correlation. If the drop of the wealth that triggered the complete liquidation is from the drop of the price of the stock with the higher expected return, then the investor likely experiences an increase in the other stock with a lower expected return and thus it is more likely that the investor has a gain in the second stock at the liquidation of the entire position of the second stock. This drives the disposition effect.

Kumar (2009) investigates stock level determinants of the disposition effect and find that the disposition effect is stronger for stocks with higher volatility. We show that our model of portfolio rebalancing can also generate such a disposition effect pattern. The main intuition is that as volatility increases, the trading boundaries are reached more frequently and thus
gains are realized more often, but the loss realizations are impacted less because when the sell boundary is reached, the investor more likely experienced a gain.

Overall, we find that our model can generate almost all of the disposition effect patterns and can match the magnitude found in the empirical literature. Obviously, this does not imply that portfolio rebalancing is the only driving force. However, our analysis suggests that in empirical investigations, one needs to separate portfolio rebalancing motivation before attributing to other potential justifications. How important is portfolio rebalancing in driving disposition effect is an important empirical question.

The remainder of the paper proceeds as follows. We first present the main model and theoretical analysis in the next section. In Section 3 we numerically solve the model and conduct simulations to illustrate that our model can generate disposition effect that also matches the empirically found magnitude. In Section 4 we extend the model to include committed consumption to show that disposition effect can also appear when an investor sells the entire position of a stock. We conclude in Section 5. All proofs are in Appendix B. In Appendix B, we show that using CRRA preferences do not change our results.

2 The Model

We consider the optimal investment problem of an investor who maximizes the expected constant absolute risk averse (CARA) utility from the final wealth at time $T$. We assume the investor can invest in one risk free money market account growing at a constant rate of $r > 0$ and $N$ risky assets (stocks). The $i$th stock price $S_{it}$ follows

$$
\frac{dS_{it}}{S_{it}} = (\mu_{0i} + \mu_{1i}Z_{it})dt + \sigma_{Si}dB_{S_{it}}^{S} \\
dZ_{it} = (g_{0i} + g_{1i}Z_{it})dt + \sigma_{Zi}dB_{Z_{it}}^{Z}
$$

where $Z_{it}$ is the predictive variable of the $i$th stock’s return, $B_{t}^{S} = (B_{1t}^{S}, ..., B_{Nt}^{S})'$, $B_{t}^{Z} = (B_{1t}^{Z}, ..., B_{Nt}^{Z})'$ are two standard $N$ dimensional Brownian motions with pairwise correlation $E[B_{it}^{S}dB_{it}^{Z}] = \rho_{i}dt$, $i = 1, ..., N$. Trading the $i$th stock incurs a fixed transaction cost of $F_{i} \geq 0$.

We use CARA preferences as our main model because with CARA preferences the portfolio rebalancing problem with multiple illiquid stocks can be reduced to individual stock rebalancing problem, which greatly simplifies computation and simulation of the optimal trading strategies. We show in the Appendix that our qualitative results stay the same when we use constant relative risk averse (CRRA) preferences.
Let $Y_i$, $i = 1, \ldots, N$, be the dollar amount invested in $i$th stock, and $X_t$ be the dollar amount invested in the money market account. Then between trading times we have

$$dX_t = rX_t dt,$$

$$dY_{it} = (\mu_{0i} + \mu_{1i}Z_{it})Y_{it} dt + \sigma_{si} Y_{it} dB_{it},$$

for $t \in [\tau_{ji}^i, \tau_{ji+1}^i)$, where $\tau_{ji}^i$ is the $j$th trading time of the $i$th stock. At $\tau_{ji}^i$, we have

$$X_{\tau_{ji}^i} = X_{\tau_{ji}^i -} - \delta_{ji}^i - F_i,$$

$$Y_{\tau_{ji}^i} = Y_{\tau_{ji}^i -} + \delta_{ji}^i,$$

where $\delta_{ji}^i$ is the dollar amount of the purchase or sale of the $i$th stock.

The investor chooses her optimal policy $\{ (\tau_i, \delta_i) : i = 1, \ldots, N \}$ to maximize

$$E[u(W_T)],$$

subject to Equations (1)-(4), where

$$u(W) = -e^{-\beta W},$$

$\beta > 0$ is the absolute risk aversion coefficient, $\tau_i = \{ \tau_{ji}^i \}$, $\delta_i = \{ \delta_{ji}^i \}$

$$W_t = X_t + \sum_{i=1}^N (Y_{it} - F_i)^+$$

is the time $t$ wealth, given the option of liquidation.

For notational simplicity, we denote by $Y_t = (Y_{1t}, \ldots, Y_{Nt})$, $Z_t = (Z_{1t}, \ldots, Z_{Nt})$, $y = (y_1, \ldots, y_N)$, $z = (z_1, \ldots, z_N)$, and let $V_1(t, x, y, z)$ be the running indirect utility function

$$V_1(t, x, y, z) = \sup_{\tau_i, \delta_i : 1 \leq i \leq N} E[u(W_T)|X_t = x, Y_t = y, Z_t = z]$$

In addition, we define an operator $\mathcal{A}$ acting on a portfolio $(x, y)$ as

$$\mathcal{A}^\delta_{j_1, \ldots, j_n}(x, y) = \left( x - \sum_{k=1}^n (\delta_{jk} + F_{jk}), \ldots, y_{j_1} + \delta_{j_1}, \ldots, y_{j_n} + \delta_{j_n}, \ldots \right)$$
In other words, $A_{j_1, \ldots, j_n}^k(x, y)$ is the investor’s new portfolio after purchasing/selling $\delta_{jk}$ dollars of the $j_k$th stock, $k = 1, \ldots, n$. Then the Hamilton-Jacobi-Bellman variational inequality can be written as

$$
\frac{\partial V_1}{\partial t} + \mathcal{L}_1 V_1 \leq 0 \quad (6)
$$

and for any $1 \leq j_1 < \ldots < j_n \leq N, n \leq N$

$$
V_1(t, x, y, z) \geq \sup_{\delta_{j_1, \ldots, j_n} \in \mathbb{R}^n} V_1(t, A_{j_1, \ldots, j_n}^k(x, y), z), \quad (7)
$$

where

$$
\mathcal{L}_1 V_1 = rx \frac{\partial V_1}{\partial x} + \sum_{i=1}^N (\mu_{0i} + \mu_{1i} z_i) y_i \frac{\partial V_1}{\partial y_i} + \frac{1}{2} \sum_{i=1}^N \sigma_{S_i}^2 y_i^2 \frac{\partial^2 V_1}{\partial y_i^2} + \sum_{i=1}^N (g_{0i} + g_{1i} z_i) \frac{\partial V_1}{\partial z_i} + \frac{1}{2} \sum_{i=1}^N \sigma_{Z_i}^2 y_i^2 \frac{\partial^2 V_1}{\partial y_i^2} + \frac{1}{2} \sum_{i=1}^N \rho_{i} \sigma_{S_i} \sigma_{Z_i} y_i \frac{\partial^2 V_1}{\partial y_i \partial z_i}.
$$

In addition, if (6) holds with strict inequality, then there exists a set of integers $\{j_1, \ldots, j_n\}$ such that (7) holds with equality. The financial interpretation is that the investor should trade stocks which are indexed by $j_1, \ldots, j_n$.

Finally, the model is completed by the terminal condition

$$
V_1(T, x, y, z) = -e^{-\beta \sum_i y_i + \sum_i (y_i - F_i)^+}. \quad (8)
$$

As in Liu (2004), due to the independence across all the stocks, the solution can be constructed for each stock as follows.

**Proposition 2.1:** Assume the function $\varphi_i(t, y_i, z_i)$ satisfies the following LCP

$$
\left\{ \begin{array}{l}
\frac{\partial \varphi_i}{\partial t} + M_i \varphi_i \leq 0, \\
\varphi_i(t, y_i, z_i) \geq \sup_{\delta_i \in \mathbb{R}} \{ \varphi_i(t, y_i + \delta_i, z_i) - \beta(\delta_i + F_i)e^{r(T-t)} \}, \\
\varphi_i(T, y_i, z_i) = \beta(y_i - F_i)^+
\end{array} \right. \quad (9)
$$

where at least one of the above inequalities holds with equality.
\[ M_i \varphi_i = (\mu_0 + \mu_i z_i) y_i \frac{\partial \varphi_i}{\partial y_i} + \frac{1}{2} \sigma_i \delta_i + \left( \frac{\partial^2 \varphi_i}{\partial y_i^2} - \left( \frac{\partial \varphi_i}{\partial y_i} \right)^2 \right) + (g_{0i} + g_{1i} z_i) \frac{\partial \varphi_i}{\partial z_i} + \frac{1}{2} \sigma_i^2 \left( \frac{\partial^2 \varphi_i}{\partial z_i^2} - \left( \frac{\partial \varphi_i}{\partial z_i} \right)^2 \right) + \rho_i \sigma_i \delta_i z_i y_i \left[ \frac{\partial^2 \varphi_i}{\partial y_i \partial z_i} - \frac{\partial \varphi_i}{\partial y_i} \frac{\partial \varphi_i}{\partial z_i} \right] \]

Then we have

\[ V_1(t, x, y, z) = -e^{-\beta x e^{r(T-t)} - \sum_{i=1}^N \varphi_i(t, y_i, z_i)}. \]

We also have the following verification theorem:

**Proposition 2.2:** Assume the function \( \varphi_i(t, y_i, z_i) \) satisfies (9), and denote \( NT_i(t) = \{(y_i, z_i) : -\frac{\partial \varphi_i}{\partial t} - M_i \varphi_i = 0\} \). Assume \( \varphi_i \) is \( C^1 \) in \( t \), and \( C^2 \) in \( (y_i, z_i) \in NT_i(t) \). Let \( \tau_i^0 = t \) and define a sequence of stopping times \( t \leq \tau_i^1 < \ldots < \tau_i^k < \ldots \) where \( \tau_i^j = \inf\{s \geq \tau_i^{j-1} : (Y_{is}, Z_{is}) \notin \text{int}(NT_i(s))\} \). In addition, define \( \delta_{\tau_i^j} \) as

\[ \delta_{\tau_i} = \arg\max_{\delta_i \in \mathbb{R}} \{ \varphi_i(\tau_i^j, Y_{\tau_i} + \delta_i, Z_{\tau_i}) - \beta(\delta_i + F_i)e^{r(T-t)} \}. \]

Then

\[ V_1(t, x, y, z) = -e^{-\beta x e^{r(T-t)} - \sum_{i=1}^N \varphi_i(t, y_i, z_i)} \]

is the value function and \((\tau_i, \delta_{\tau_i})\) is the optimal trading policy for the \( i \)th stock, \( i = 1, \ldots, N \), where \( \tau_i = \{\tau_i^j\} \) and \( \delta_{\tau_i} = \{\delta_{\tau_i^j}\} \), for \( j = 0, 1, 2, \ldots \).

### 3 Numerical Results

In this section, we numerically solve the investor’s portfolio rebalancing problem to obtain the optimal trading strategies and conduct Monte Carlo simulation to show that our model can generate the widely documented disposition effect.

We use the penalty method (cf. Forsyth and Vetzal (2002), and Dai and Zhong (2010)) to numerically solve (9). The penalty equation is given by

\[
\begin{cases}
\frac{\partial \varphi_i}{\partial t} + M_i \varphi_i + \lambda \left( \sup_{\delta_i \in \mathbb{R}} \{ \varphi_i(t, y_i + \delta_i, z_i) - \beta(\delta_i + F_i)e^{r(T-t)} \} - \varphi_i(t, y_i, z_i) \right)^+ = 0, \\
\varphi_i(T, y_i, z_i) = \beta(y_i - F_i)^+
\end{cases}
\]

(10)

where \( \lambda \) is a large penalty parameter, and \( a^+ = \max\{a, 0\} \). As \( \lambda \to \infty \), the solution to Equation (10) converges to the solution to Inequalities (9).
Table 1: Baseline Parameter Values in the CARA Model.

This table summarizes the baseline parameter values we use to illustrate our results. For simplicity, all stocks are assumed to be homogeneous, i.e., their structural parameters are assumed to have the same values.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Baseline value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk free rate</td>
<td>$r$</td>
<td>0.01</td>
</tr>
<tr>
<td>Long term average return</td>
<td>$\mu_0$</td>
<td>0.08</td>
</tr>
<tr>
<td>Loading of the predictor</td>
<td>$\mu_1$</td>
<td>1.2</td>
</tr>
<tr>
<td>Volatility of the stock</td>
<td>$\sigma_S$</td>
<td>0.2</td>
</tr>
<tr>
<td>Average predictor value</td>
<td>$g_0$</td>
<td>0.0</td>
</tr>
<tr>
<td>(Negative) reverting speed of the predictor</td>
<td>$g_1$</td>
<td>-0.5</td>
</tr>
<tr>
<td>Volatility of the predictor</td>
<td>$\sigma_Z$</td>
<td>0.015</td>
</tr>
<tr>
<td>Correlation between shocks in stock return</td>
<td>$\rho$</td>
<td>0.5</td>
</tr>
<tr>
<td>and shocks in the predictor</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of risky stocks</td>
<td>$N$</td>
<td>10</td>
</tr>
<tr>
<td>Fixed transaction cost</td>
<td>$F_i$</td>
<td>$1$</td>
</tr>
<tr>
<td>Investment horizon</td>
<td>$T$</td>
<td>5 years</td>
</tr>
<tr>
<td>Absolute risk aversion</td>
<td>$\beta$</td>
<td>0.001</td>
</tr>
</tbody>
</table>

We report the baseline parameter values in Table 1. The results for a large set of other parameter values are similar. For simplicity, we assume there are ten stocks that the investor can invest in, all the stocks have the same long term expected return of 8% and volatility of 20%, their predictors have long term average value of 0.0, mean reverting speed of 0.5, and volatility of 1.5%. The correlation between the shocks on stock returns and the shocks on the predictors is assumed to be 0.5. We also assume interest rate is 1%, fixed trading cost is $1, risk aversion coefficient is 0.001, and the investment horizon is five years. Because of the well known absence of the wealth effect, the level of initial wealth is not important for the numerical results.

3.1 Optimal Trading Policies

We first plot the optimal trading strategy in Figure 1 to illustrate how the investor trades in a stock.\(^4\) Figure 1 shows that there exists a no-transaction region within which it is optimal not to trade. When the dollar value of the stock position rises to the sell boundary or decreases to the buy boundary, then the investor trades vertically to the target level, represented by the middle thick line. In addition, as the predictive state variable increases,

\(^4\) Because given the default parameter values, all the stocks have the same expected return and volatility, the trading strategies are the same across all the stocks.
Figure 1: Optimal buying and selling boundary of a single stock with CARA utility and return predictability, $t = 0$. Parameter values: $r = 0.01$, $\mu_{0i} = 0.08$, $\mu_{1i} = 1.20$, $\sigma_{Si} = 0.20$, $g_{0i} = 0.00$, $g_{1i} = -0.50$, $\sigma_{Zi} = 0.015$, $\rho_{i} = 0.50$, $F_{i} = $1, $T = 5$, $\beta = 0.001$.

the expected return of the stock increases, the investor on average invests more in the stock, and therefore the no-transaction region shifts upward.

### 3.2 Disposition Effect

To see whether the widely documented disposition effect is consistent with our model, we conduct simulations of the optimal trading strategies implied by the model. Following Odean (1998), each day that a sale takes place, we compare the selling price for each stock sold to its average purchase price to determine whether that stock is sold for a gain or a loss. Each stock that is in that portfolio at the beginning of that day, but is not sold, is considered to be a paper (unrealized) gain or loss (or neither). Whether it is a paper gain or loss is determined by comparing its highest and lowest price for that day to its average purchase price. If both its daily high and low are above its average purchase price it is counted as a paper gain; if they are both below its average purchase price it is counted as a paper loss; if its average purchase price lies between the high and the low, neither a gain or loss is counted. On days when no sales take place in an account, no gains or losses, realized or paper, are counted. For each simulated path, using the above definitions, we record the number of realized gains/losses (# Realized Gains/Losses) and the number of paper gains/losses (# Paper Gains/Losses) over the entire investment horizon $[0, T]$ for the optimal trading strategy.
Then we calculate the following ratios as used by Odean (1998):

\[
PGR = \frac{\text{#Realized Gains}}{\text{#Realized Gains} + \text{#Paper Gains}}
\]

\[
PLR = \frac{\text{#Realized Losses}}{\text{#Realized Losses} + \text{#Paper Losses}}
\]

We also compute the fraction of sales that are gains, i.e.,

\[
PGL = \frac{\text{#Realized Gains}}{\text{#Realized Gains} + \text{#Realized Losses}}
\]

These quantities are then averaged over all the simulated paths, and their mean are reported in Table 2. Table 2 shows that our model can indeed generate the disposition effect documented in the existing literature. For example, in Table 1 of Odean (1998), he reports PLR of 0.098 and PGR of 0.148. In comparison, our model implies PLR of 0.0348 and PGR of 0.1311, with very small standard errors. The disposition effect \( DE = PLR - PGR \) is equal to \(-0.0963\) and statistically significant. In addition, among all the sales, gains realizations account for more than 88%. The main intuition for our results is as follows. In the absence of transaction costs, it is optimal for the investor to invest a constant dollar amount in a stock, which implies that when the stock price rises the investor sells and when the stock price drops the investor buys. This clearly implies that the investor realizes gains more often than losses. However, due to the absence of transaction costs, the investor trades continuously and the optimal trading strategy cannot match the empirical findings. With the fixed transaction costs, however, the investor optimally trade infrequently and every time the investor trades, a lump sum transaction is made, as observed in empirical evidence. On the other hand, the presence of transaction costs does not change the qualitative conclusion that to keep the risk exposure within a range, the investor still realizes gains more often than losses. This is because stocks that are bought have positive expected returns and thus it is more likely the investor has a gain when the sell boundary is reached after an increase in the stock price. In contrast, after a decrease in the stock price, the buy boundary is more likely to be reached than the sell boundary. This suggests that it is less likely that when the sell boundary is reached, the investor has a loss. This generates the trading pattern that is consistent with the disposition effect.

When there is only one stock in a portfolio, clearly both PGR and PLR are equal to one, because conditional on a sale, it is either a realized gain or a realized loss, but there is no paper gain or paper loss from another stock. As the number of stocks in a portfolio increases, the PGR and PLR ratios tend to become smaller, because with the fixed transaction cost and
the independence of returns, when one stock reaches its sell boundary, it is likely that many other stocks are still in the no-transaction regions and thus only have paper gains or paper losses. Therefore one can vary the number of the stocks held in a portfolio to match the empirically found magnitude of the disposition effect. Figure 2 shows how the magnitudes of the ratios PLR, PGR and the disposition effect DE monotonically vary as we increase the number of stocks in a portfolio.\footnote{We also used alternative disposition effect measures used in the literature (e.g., Odean (1998)) and obtained similar results.}

Because in standard portfolio rebalancing models without transaction costs (e.g., Merton (1972)), it is never optimal for an investor to sell a stock without purchasing some other stocks, Odean (1998) shows that among the sales after which there were no purchases in three weeks, the disposition effect still appears and thus conclude that the disposition effect can unlikely explain the disposition effect in this subsample. However, in the presence of fixed transaction costs, it can be optimal for an investor to sell a stock without purchasing another for some time. Because as long as other stocks are inside their no-transaction regions, it is not optimal for the investor to buy any additional amount of other stocks. To see if our model can generate the disposition effect conditional on there are no purchases of any other stocks within three weeks after a sale of stock, we compute the PLR and PGR ratios conditional on these subsample paths. We report the results in the right most column of

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Disposition effect measures and the number of stocks in a portfolio. Parameter values: $r = 0.01$, $\mu_{0i} = 0.08$, $\mu_{1i} = 1.20$, $\sigma_{Si} = 0.20$, $g_{0i} = 0.00$, $g_{1i} = -0.50$, $\sigma_{Zi} = 0.015$, $\rho_i = 0.50$, $F_i = \$1$, $i = 1, \ldots, N$, $T = 5$, $\beta = 0.001$.}
\end{figure}
Table 2. The results are very similar to the results obtained for the entire sample. This shows that the disposition effect found in the subsample Odean (1998) considered can also be consistent with a portfolio rebalancing model like ours.

Kumar (2009) investigates stock level determinants of the disposition effect and find that the disposition effect is stronger for stocks with higher volatility. Kumar argues that this is consistent with the disposition effect being stronger among stocks that are more difficult to value and behavioral biases being stronger for such stocks. Table 3 shows that when the volatility is increased to 30% the disposition effect also increases in our model. Therefore, portfolio rebalancing can also contribute to this disposition effect pattern. The main intuition is that as volatility increases, the trading boundaries are reached more frequently and thus gains are realized more often and the loss realizations are impacted less. For example, the second column of Table 3 shows when the volatility increases to 30%, PGR increases by 0.0286, while PLR decreases by 0.0208, which leads to a significant increase in the disposition effect measure.

It has been widely documented that investors tend to sell winners that subsequently outperform held (unrealized) losers (e.g., Odean (1998)). The existing literature argues that this behavior indicates that investors tend to sell winners too soon and hold losers too long, and thus should sell winners later and sell losers sooner. We report the average ex post returns of stocks sold as winners and of stocks held as losers from simulations of our model in Table 4. Table 4 shows that the optimal portfolio rebalancing can produce the trading pattern that sold winners subsequently outperform held losers. The intuition is that if the expected return of a stock increases with a predictive state variable that is positively correlated to the stock return, then conditional on a positive shock to the stock return, the average level of the predictive state variable is greater than that conditional on a negative return. Therefore, the expected return of a winner can subsequently outperform the expected return of a loser. However, to achieve optimal risk exposure, it can still be optimal to sell winners and to hold losers due to transaction costs.

The left subfigure of Figure 3 shows how the average time to realize a loss changes with transaction costs. As transaction costs increases, the delay of loss realization increases, due to the expanded no transaction region. The right subfigure of Figure 3 shows how the average return from the time a loss occurs to the time the loss is realized changes with transaction costs. As transaction costs increases, the average return become more negative. This is because with a lower expected return, the longer the delay, the lower the realized return on average.
Table 2: Disposition effect

This table shows the average disposition effect measures. The results are obtained from 10,000 paths of Monte Carlo simulation in the model with CARA utility, ten independent stocks and ten independent return predictors. Parameter values: \( r = 0.01, \mu_{0i} = 0.08, \mu_{1i} = 1.20, \sigma_{Si} = 0.20, g_{0i} = 0.00, g_{1i} = -0.50, \sigma_{Z_i} = 0.015, \rho_{i} = 0.50, F_{i} = \$1, i = 1, \ldots, 10, T = 5, \beta = 0.001. \)

<table>
<thead>
<tr>
<th>Entire trading history</th>
<th>No new purchase in the following 3 weeks</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLR</td>
<td>0.0348</td>
</tr>
<tr>
<td>PGR</td>
<td>0.1311</td>
</tr>
<tr>
<td>DE</td>
<td>-0.0963 (0.0004)</td>
</tr>
<tr>
<td>PGL</td>
<td>0.8882</td>
</tr>
</tbody>
</table>

Table 3: Disposition effect and volatility

This table shows the average disposition effect measures with two different volatility levels. The results are obtained from 10,000 paths of Monte Carlo simulation in the model with CARA utility, ten independent stocks and ten independent return predictors. Parameter values: \( r = 0.01, \mu_{0i} = 0.08, \mu_{1i} = 1.20, \sigma_{Si} = 0.20 \) or 0.30, \( g_{0i} = 0.00, g_{1i} = -0.50, \sigma_{Z_i} = 0.015, \rho_{i} = 0.50, F_{i} = \$1, i = 1, \ldots, 10, T = 5, \beta = 0.001. \)

<table>
<thead>
<tr>
<th>Volatility level ( \sigma_{S} )</th>
<th>0.20</th>
<th>0.30</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLR</td>
<td>0.0348</td>
<td>0.0140</td>
</tr>
<tr>
<td>PGR</td>
<td>0.1311</td>
<td>0.1597</td>
</tr>
<tr>
<td>DE</td>
<td>-0.0963 (0.0004)</td>
<td>-0.1457 (0.0004)</td>
</tr>
<tr>
<td>( \Delta \text{(DE)} )</td>
<td>-0.0494 (0.0006)</td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Ex post returns

This table shows the average ex post returns of the stocks sold as winners and of the stocks held as losers. The results are obtained from 10,000 paths of Monte Carlo simulation in the model with CARA utility, ten independent stocks and ten independent return predictors. Parameter values: \( r = 0.01, \mu_{0i} = 0.08, \mu_{1i} = 1.20, \sigma_{Si} = 0.20, g_{0i} = 0.00, g_{1i} = -0.50, \sigma_{Z_i} = 0.015, \rho_{i} = 0.50, F_{i} = \$1, i = 1, \ldots, 10, T = 5, \beta = 0.001. \)

<table>
<thead>
<tr>
<th></th>
<th>Return over the next 6 months</th>
<th>Return over the next 12 months</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stocks sold as winners</td>
<td>4.32%</td>
<td>8.63%</td>
</tr>
<tr>
<td>Stocks held as losers</td>
<td>3.64%</td>
<td>7.59%</td>
</tr>
<tr>
<td>Difference</td>
<td>0.68% (0.06%)</td>
<td>1.04% (0.10%)</td>
</tr>
</tbody>
</table>
Figure 3: The average time to realize a loss and the average return of the realized losses. Parameter values: \( r = 0.01, \mu_0 = 0.08, \mu_1 = 1.20, \sigma_{s_i} = 0.20, g_{0i} = 0.00, g_{1i} = -0.50, \sigma_{z_i} = 0.015, \rho_i = 0.50, i = 1, \ldots, 10, \alpha = 5, \beta = 0.001. \)

4 Disposition Effect in the Subsample with Total Liquidation

Odean (1998) also shows that in the subsample where investors sell the entire position there is still disposition effect. He argues that an investor would not liquidate the entire position of a stock for rebalancing purposes and therefore the disposition effect in this subsample cannot be caused by portfolio rebalancing. We next extend our portfolio rebalancing model to show that liquidating the entire position in a stock can be also caused by portfolio rebalancing and in addition, there can be disposition effect in the subsample with total liquidation in such a portfolio rebalancing model.

It is widely documented that a large portion of the average households budget is committed to ensure a certain critical level of consumption (e.g., Fratantoni (2001), Chetty and Szeidl (2007)). Committed consumption can be caused by sources such as housing and other durable goods consumption that is costly to adjust, habit formation, meeting fixed financial obligations (e.g., mortgage and tuition payments), and precautionary savings against unemployment or health shocks. In addition, investors rarely short stocks or borrow to buy stocks. Accordingly, we assume that the investor must invest at least \( Ce^{-r(T-t)} \) in the risk
free asset at time $t$ to ensure that the terminal wealth at time $T$ is at least a minimum level of $C$.  

To understand why it can be optimal to liquidate the entire position in a stock and to display the disposition effect in a portfolio rebalancing model, in this section we focus on the simplest case where there are only two stocks $S_1$ and $S_2$.  

$S_1$ is assumed to be perfectly liquid so that $F_1 = 0$. In addition, in order to maintain the tractability, we drop the predictors in the stock returns, so that $\mu_{1i} = 0$, $i = 1, 2$. For notational simplicity, we denote by $\mu_1 = \mu_{01}$, $\mu_2 = \mu_{02}$, $\sigma_1 = \sigma_{S1}$, $\sigma_2 = \sigma_{S2}$ and $F = F_2$. The two Brownian motions $B_{1t}$ and $B_{2t}$ are allowed to have correlation $\rho \in [-1, 1]$.

Again, we denote by $X_t, Y_t$ the time $t$ dollar amount invested in the liquid assets (liquid stock plus the risk free asset) and in the illiquid stock, respectively. Let $0 \leq \tau_1 < \tau_2 < \cdots < \tau_n < ... \leq T$ be the time points when $S_2$ is traded. Then we have

$$
dX_t = rX_t dt + \xi_t(\mu_1 - r)dt + \xi_t\sigma_1 dB_{1t},
$$

$$
dY_t = \mu_2 Y_t dt + \sigma_2 Y_t dB_{2t},
$$

for $t \in [\tau_i, \tau_{i+1})$, where $\xi_t$ represents the dollar amount invested in the liquid stock at time $t$. At the transaction time $\tau_i$, we have

$$
X_{\tau_i} = X_{\tau_i^-} - \delta_{\tau_i} - F,
$$

$$
Y_{\tau_i} = Y_{\tau_i^-} + \delta_{\tau_i},
$$

where $\delta_{\tau_i}$ represents the change of the dollar value invested in the illiquid stock.

We assume no short-selling or borrowing and the investor is required to be solvent at all time. Thus we have the following constraints:

$$
X_t - \xi_t \geq Ce^{-r(T-t)}, \quad \xi_t \geq 0, \quad Y_t \geq 0.
$$

---

6 We view a continuous-time model as an approximation of discrete-time trading in practice. As Liu (2014) shows, with discrete-time trading and a solvency constraint, an investor must invest at least the present value of the committed consumption level in the risk free asset.

7 To match the magnitude of the disposition effect, one needs to extend to the case with multiple illiquid stocks as we did in the previous section. With the additional committed consumption constraint, it is no longer feasible to separate the investor’s multi-stock rebalancing problem into individual stock rebalancing problems. Therefore, one needs to solve numerically a multi-dimensional optimal impulse control problem, which is much more time consuming than the model without the constraint.

8 Even when $y_t < F$, the investor can hold her stock position and the solvency will not be breached.
Given an initial endowment \((X_0, Y_0)\) that satisfies the solvency constraints, the investor’s objective is to choose \(\{\tau, \delta, \xi\}\) to maximize the expected utility \(E[u(W_T)]\) derived from her wealth at time \(T\), subject to Equations (11)-(14) and the solvency constraints (15).

### 4.1 The HJB Equation and Verification Theorem

Let \(V_2(x, y, t)\) be the value function of the investor’s problem in this case. The HJB equation that governs the investor’s value function is

\[
\min \left\{ -\frac{\partial V_2}{\partial t} - \mathcal{L}_2 V_2, \ V_2(x, y, t) - f(x, y, t) \right\} = 0, \ t < T
\]

where

\[
\mathcal{L}_2 V_2 = \frac{1}{2} \sigma_y^2 y^2 \frac{\partial^2 V_2}{\partial y^2} + \mu_y y \frac{\partial V_2}{\partial y} + r x \frac{\partial V_2}{\partial x} + \max_{\xi \in [0, x - Ce^{-r(T-t)}]} \left\{ \frac{1}{2} \sigma_x^2 \xi^2 \frac{\partial^2 V_2}{\partial x^2} + (\mu_1 - r) \xi \frac{\partial V_2}{\partial x} + \rho \sigma_x \xi y \frac{\partial^2 V_2}{\partial x \partial y} \right\}
\]

and

\[
f(x, y, t) = \begin{cases} -\infty, & C e^{-r(T-t)} \leq x + y < F + C e^{-r(T-t)}, \\ \sup_{\theta \in I} V_2(x - \delta - F, y + \delta, t), & x + y \geq F + C e^{-r(T-t)} \end{cases}
\]

for \(I = [-y, x - Ce^{-r(T-t)} - F] \setminus \{0\}\).\(^9\)

When \(x + y = Ce^{-r(T-t)}\), the investor can only derive utility through the committed consumption \(C\) at \(T\), which leads to the following boundary condition

\[
V_2(x, y, t)|_{x+y=Ce^{-r(T-t)}} = u(C)
\]

(17)

The model is completed by the terminal condition

\[
V_2(x, y, T) = u(x + (y - F)^+)
\]

(18)

We next provide a verification theorem for the optimal trading strategies.

**Proposition 4.1:** Let \(\phi(x, y, t)\) be the solution of HJB equation (16) and denote \(NT(t) = \{(x, y) \in \Psi : -\frac{\partial \phi}{\partial t} - \mathcal{L}_2 \phi = 0\}\). Assume \(\phi\) is \(C^1\) in \(t\), and \(C^2\) in \((x, y) \in NT(t)\). Let \(\tau_0 = t\) and define a sequence of stopping times \(t \leq \tau_1 < ... < \tau_k < ... < T\) as \(\tau_i = \inf\{s \geq \tau_{i-1} : (X_s, Y_s) \notin \text{int}(NT(s))\}\). In addition, define

\[
\delta_{\tau_i} = \arg \max_{\delta \in I_{\tau_i}} \phi(X_{\tau_i} - \delta - F, Y_{\tau_i} + \delta, \tau_i)
\]

\(^9\) Note that that in the region \(D_1 = \{(x, y) \in \mathbb{R}_+^2 : Ce^{-r(T-t)} \leq x + y < F + Ce^{-r(T-t)}\}\), no admissible trading strategy exists since the net wealth after any transaction will be below \(Ce^{-r(T-t)}\).
Table 5: Baseline Parameter Values in the CARA Model with Committed Consumption.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Baseline value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk free rate</td>
<td>r</td>
<td>0.01</td>
</tr>
<tr>
<td>Expected return of liquid stock</td>
<td>$\mu_1$</td>
<td>0.09</td>
</tr>
<tr>
<td>Volatility of liquid stock</td>
<td>$\sigma_1$</td>
<td>0.20</td>
</tr>
<tr>
<td>Expected return of illiquid stock</td>
<td>$\mu_2$</td>
<td>0.07</td>
</tr>
<tr>
<td>Volatility of illiquid stock</td>
<td>$\sigma_2$</td>
<td>0.15</td>
</tr>
<tr>
<td>Correlation between the returns</td>
<td>$\rho$</td>
<td>0</td>
</tr>
<tr>
<td>Committed consumption level</td>
<td>$C$</td>
<td>$100,000$</td>
</tr>
<tr>
<td>Fixed transaction cost</td>
<td>$F$</td>
<td>$5$</td>
</tr>
<tr>
<td>Investment horizon</td>
<td>$T$</td>
<td>5 years</td>
</tr>
<tr>
<td>Absolute risk aversion</td>
<td>$\beta$</td>
<td>0.00002</td>
</tr>
</tbody>
</table>

where $I_{\tau_i} = [-Y_{\tau_i}, X_{\tau_i} - Ce^{-r(T-\tau_i)} - F] \setminus \{0\}$, and

$$
\xi_t = \arg \max_{\xi \in [0, X_t - Ce^{-r(T-t)}]} \left\{ \frac{1}{2} \sigma_1^2 \xi^2 \frac{\partial^2 \phi}{\partial x^2} + (\mu_1 - r) \xi \frac{\partial \phi}{\partial x} + \rho \sigma_1 \sigma_2 \xi Y_t \frac{\partial^2 \phi}{\partial x \partial y} \right\}
$$

where all the partial derivatives are evaluated at $(X_t, Y_t, t)$. Then $\phi(x, y, t)$ is the value function and $((\tau_i, \delta_{\tau_i}), \xi_t)$ is the optimal trading policy.

### 4.2 Numerical Results

For simplicity, we assume that the liquid stock has an expected return of 9% and volatility of 20%, the illiquid stock has an expected return of 7% and volatility of 15%, the correlation coefficient between the two stocks is 0, interest rate is 1%, fixed trading cost is $5, the committed consumption level $C = $100,000, the absolute risk aversion coefficient is 0.00002 (implying a relative risk aversion coefficient of 2 at a wealth level of $100,000), and the investment horizon is five years. We report these baseline parameter values in Table 5.

#### 4.2.1 Optimal Trading Policies

We plot the optimal trading policy at time 0 in Figure 4. For ease of exposition, we always measure the portfolio weight as the weight in the disposable wealth, i.e., $\frac{y}{x + y - Ce^{-r(T-t)}}$. When the portfolio weight in the illiquid stock exceeds the selling boundary, the investor sell a lump sum amount of the stock so that the portfolio weight in the stock drops to the middle target ratio line. When the portfolio weight drops below the buying boundary, the investor buys a lump sum amount so that the portfolio weight increases to the target ratio line.
Trading direction is marked on the figure, and note that the total liquidation of the illiquid stock is possible when the disposable wealth is small. Intuitively, this is because when the disposable wealth is low, the investment in risky asset is small and thus the expected return dominates return risk and the investor optimally chooses to hold only the stock with the highest expected return, which is the liquid stock.

4.2.2 Disposition Effect

We now conduct the same analysis as before by computing the ratios PLR, PGR, DE, and PGL using Monte Carlo simulations. We report the results in Table 6. Table 6 shows that the disposition effect exists across a large range of wealth levels. For example, at disposable wealth to the committed consumption ratio of 0.1, the PGR is equal to 0.560, while PLR is only 0.329. In addition, in all the sales, more than 84% are with a gain. The smallest disposition effect of DE= \(0.423 - 0.532 = -0.110\) occurs when the disposable wealth to the committed consumption ratio is 0.15. The disposition effect is strongly statistically significant. The lowest PGL is still as high as 82.1%. In addition, our model suggests that as the number of stocks in a portfolio decreases, the disposition effect gets even stronger. This is because the number of paper gains tend to decrease more than the number of paper losses when the expected returns are positive.
Table 6: Entire trading history

This table shows the results obtained from 100,000 simulated paths, with various initial disposable wealth \( w_0 \) (normalized by the committed consumption \( C \)). We show the results of mean and standard deviation (in the parenthesis). Parameter values: \( r = 0.01, \mu_1 = 0.09, \sigma_1 = 0.20, \mu_2 = 0.07, \sigma_2 = 0.15, \rho = 0.0, C = \$100,000, F = \$5, T = 5, \beta = 0.00002 \). The variable “DE” records the value of PLR-PGR.

\[
\begin{array}{cccccccccc}
  \frac{w_0}{C} & 0.1 & 0.15 & 0.2 & 0.25 & 0.3 & 0.4 & 0.5 & 0.75 & 1 & 1.5 \\
  \text{PLR} & 0.329 & 0.423 & 0.249 & 0.188 & 0.165 & 0.144 & 0.158 & 0.283 & 0.185 & 0.383 \\
  & (0.009) & (0.004) & (0.002) & (0.001) & (0.001) & (0.001) & (0.001) & (0.001) & (0.001) & (0.001) \\
  \text{PGR} & 0.560 & 0.532 & 0.663 & 0.686 & 0.681 & 0.667 & 0.654 & 0.612 & 0.622 & 0.567 \\
  & (0.004) & (0.002) & (0.001) & (0.001) & (0.001) & (0.001) & (0.001) & (0.000) & (0.000) & (0.000) \\
  \text{DE} & -0.231 & -0.110 & -0.414 & -0.497 & -0.516 & -0.523 & -0.496 & -0.329 & -0.437 & -0.184 \\
  & (0.015) & (0.006) & (0.002) & (0.002) & (0.001) & (0.001) & (0.001) & (0.001) & (0.001) & (0.001) \\
  \text{PGL} & 0.848 & 0.844 & 0.836 & 0.843 & 0.859 & 0.883 & 0.878 & 0.821 & 0.861 & 0.843 \\
  & (0.005) & (0.002) & (0.001) & (0.001) & (0.001) & (0.001) & (0.001) & (0.001) & (0.001) & (0.001) \\
\end{array}
\]

Table 7: Restricted to the sales in which the position in one stock is fully liquidated

This table shows the results obtained from 100,000 simulated paths, with various initial disposable wealth \( w_0 \) (normalized by the committed consumption \( C \)). We show the results of mean and standard deviation (in the parenthesis). Parameter values: \( r = 0.01, \mu_1 = 0.09, \sigma_1 = 0.20, \mu_2 = 0.07, \sigma_2 = 0.15, \rho = 0.0, C = \$100,000, F = \$5, T = 5, \beta = 0.00002 \). The variable “DE” records the value of PLR-PGR.

\[
\begin{array}{cccccccc}
  \frac{w_0}{C} & 0.1 & 0.15 & 0.2 & 0.25 & 0.3 & 0.4 & 0.5 \\
  \text{PLR} & 1.000 & 1.000 & 0.349 & 0.241 & 0.188 & 0.218 & 0.289 \\
  & (0.000) & (0.000) & (0.002) & (0.002) & (0.002) & (0.004) & (0.011) \\
  \text{PGR} & 0.312 & 0.339 & 0.637 & 0.837 & 0.961 & 0.999 & 1.000 \\
  & (0.005) & (0.002) & (0.002) & (0.002) & (0.002) & (0.001) & (0.000) \\
  \text{DE} & 0.688 & 0.661 & -0.289 & -0.597 & -0.773 & -0.781 & -0.711 \\
  & (0.003) & (0.002) & (0.005) & (0.004) & (0.003) & (0.002) & (0.004) \\
  \text{PGL} & 0.616 & 0.663 & 0.668 & 0.647 & 0.660 & 0.570 & 0.426 \\
  & (0.010) & (0.003) & (0.002) & (0.002) & (0.003) & (0.008) & (0.021) \\
\end{array}
\]
To see if disposition effect can still appear in the subsample where an investor sells the entire position in a stock, we next restrict to the subsample paths along which the investor liquidates the entire illiquid stock position and report the corresponding results in Table 7. Table 7 shows that indeed our portfolio rebalancing model can generate the disposition effect when the disposable wealth to the committed consumption ratio is reasonably high. For example, when the disposable wealth to the committed consumption ratio is 0.2, we find the PGR is equal to 0.637, while PLR is only 0.349, implying a statistically significant disposition effect of $-0.289$. In addition, the fraction of sales that have gains is greater than 50% in most wealth levels, except when the disposable wealth to committed consumption ratio is equal to 0.5. Intuitively, the investor liquidates the entire position of the illiquid stock when the total wealth decreases beyond a threshold and the portfolio weight in the illiquid stock exceed the sell boundary. If this wealth decrease is caused by a decline in the liquid stock and there is a gain in the illiquid stock, then the portfolio is more imbalanced and thus it is more likely that the investor liquidates the entire illiquid stock position. Compared to the case with partial sale where a sale is more likely triggered by a price increase, the disposition effect is smaller because total liquidation in our model is triggered by a stock price drop. If it is the drop in the illiquid stock price that causes the total liquidation, then the sale is a realized loss. Thus the percentage of realized losses increases.

We also note that when the initial disposable wealth is too low (0.1 or 0.15), the disposition effect disappears in the subsample of sales in which the entire position of illiquid stock is sold. The reason is that there needs to be huge gain on the liquid stock so that the investor will start to buy the illiquid stock. As a result, when the investor sells the entire position of the illiquid stock, the liquid stock is still likely to have gain. Consequently, more paper gains are recorded, and disposition effect disappears.

Since total liquidation of the illiquid stock will only occur when the disposable wealth level reaches a lower threshold, its probability decreases dramatically when the initial disposable wealth become higher. For example, when the initial disposable wealth is equal to the committed consumption level $C$, no such liquidation is observed in the simulated sample paths.

5 Conclusion

Disposition effect the investors tend to sell winners too soon and hold losers too long has been widely documented and behavioral types of explanations have dominated in the literature. As far as we know, there is no theoretical model in a rational framework proposed that can help explain disposition effect and match the empirically found magnitude. In this paper, we
develop a new optimal portfolio rebalancing model in the presence of transaction costs and committed consumption. We show that almost all of the disposition effect patterns found in the existing literature are consistent with optimal trading strategies implied by our model. In addition, our model can also produce disposition effect that matches the magnitudes found in empirical studies. Therefore, it becomes an empirical question how much disposition effect cannot be explained by optimal portfolio rebalancing.
References


Appendix A

In this Appendix, we collect the proofs of our analytical results.

Proof of Proposition 2.1:

Proof. We shall directly verify that the constructed function $V_1(t, x, y, z)$ satisfies the original HJB equation.

First we verify that $V_1$ satisfies (6). Straightforward calculation yields

$$
\frac{\partial V_1}{\partial t} + L_1 V_1 = -V_1 \sum_{i=1}^{N} \left( \frac{\partial \varphi_i}{\partial t} + M_i \varphi_i \right) \leq 0 \tag{19}
$$

since $-V_1 > 0$.

Next we verify that $V_1$ satisfies (7). For any set of integers $J = (j_1, ..., j_n)$ such that $1 \leq j_1 < ... < j_n \leq N$, we have

$$
\sum_{i \in J} \varphi_i(t, y_i, z_i) \geq \sum_{i \in J} \sup_{\delta_i \in \mathbb{R}} \left( \varphi_i(t, y_i, z_i) - \beta(\delta_i + F_i)e^{r(T-t)} \right) - \sum_{i \notin J} \varphi_i(t, y_i, z_i)
$$

Therefore

$$
V_1(t, x, y, z) \geq -e^{-\beta e^{r(T-t)} \sum_{i \notin J} \varphi_i(t, y_i, z_i)} \sum_{i \in J} \sup_{\delta_i \in \mathbb{R}} \left( \varphi_i(t, y_i, z_i) - \beta(\delta_i + F_i)e^{r(T-t)} \right)\sum_{i \notin J} \varphi_i(t, y_i, z_i)
$$

Now we verify the complementarity. Assume

$$
\frac{\partial V_1}{\partial t} + L_1 V_1 < 0
$$

then there exists $J = \{j_1, ..., j_n\}$ with $1 \leq j_1 < ... < j_n \leq N$, such that

$$
\frac{\partial \varphi_i}{\partial t} + M_i \varphi_i < 0,
$$

for $i \in J$. Therefore

$$
\varphi_i(t, y_i, z_i) = \sup_{\delta_i \in \mathbb{R}} \left\{ \varphi_i(t, y_i + \delta_i, z_i) - \beta(\delta_i + F_i)e^{r(T-t)} \right\}, \quad j \in J
$$
Consequently, similar calculation yields

\[ V_1(t, x, y, z) = \sup_{(\delta_{j_1}, \ldots, \delta_{j_n}) \in \mathbb{R}^n} V_1(t, A^\delta_{j_1, \ldots, j_n}(x, y, z)) \]

The terminal condition is clearly matched. The proof is completed. \( \square \)

**Proof of Proposition 2.2:**

*Proof.* The proof is standard for impulse control problem so we only sketch it. Let \( (s^i, \eta^i_s) \) be an arbitrary set of admissible trading strategy of stock \( i \). Without loss of generality, we assume that the trading times \( s^i \) are ordered as a strictly increasing sequence \( s_1 < s_2 < \ldots \), with trading amount \( \eta_{s_k} \) at \( s_k \).

Applying Ito’s lemma, we have

\[
E \left[ V_1(s_k, X_{s_k}, Y_{s_k}, Z_{s_k}) - V_1(s_{k+1}^-, X_{s_{k+1}^-}, Y_{s_{k+1}^-}, Z_{s_{k+1}^-}) \right] = E \left[ \int_{s_j}^{s_{j+1}} \left( -\frac{\partial V_1}{\partial t} - \mathcal{L}_1 V_1 \right)(X_s, Y_s, Z_s) ds \right] \geq 0
\]

where the inequality is due to (19). Therefore we have

\[
E \left[ V_1(s_k, X_{s_k}, Y_{s_k}, Z_{s_k}) - V_1(s_{k+1}^-, X_{s_{k+1}^-}, Y_{s_{k+1}^-}, Z_{s_{k+1}^-}) \right] \geq E \left[ V_1(s_{k+1}^-, X_{s_{k+1}^-}, Y_{s_{k+1}^-}, Z_{s_{k+1}^-}) - V_1(s_{k+1}^-, X_{s_{k+1}^-}, Y_{s_{k+1}^-}, Z_{s_{k+1}^-}) \right]
\]

Assume stock \( m \) is traded at time \( s_{k+1} \), we have

\[ V_1(s_{k+1}^-, X_{s_{k+1}^-}, Y_{s_{k+1}^-}, Z_{s_{k+1}^-}) = V_1(s_{k+1}^-, A^\delta_{m, s_{k+1}^-}(X_{s_{k+1}^-}, Y_{s_{k+1}^-}, Z_{s_{k+1}^-})) \]

Straightforward substitution of \( \varphi_i \) yields

\[ V_1(s_{k+1}^-, X_{s_{k+1}^-}, Y_{s_{k+1}^-}, Z_{s_{k+1}^-}) \leq V_1(s_{k+1}^-, X_{s_{k+1}^-}, Y_{s_{k+1}^-}, Z_{s_{k+1}^-}) \]

hence

\[ E \left[ V_1(s_k, X_{s_k}, Y_{s_k}, Z_{s_k}) - V_1(s_{k+1}^-, X_{s_{k+1}^-}, Y_{s_{k+1}^-}, Z_{s_{k+1}^-}) \right] \geq 0 \]

\[ ^{10} \text{In fact, similar to Liu (2004), the intermediate trading will involve only one stock each time, with probability one. It could be optimal to trade multiple stocks at the initial time, if the initial allocation lies in the intersection of multiple trading regions. However, this will not change the proof significantly.} \]
For a sample path $\omega$, let $M(\omega) = \max\{k \geq 1 : s_k(\omega) \leq T < s_{k+1}(\omega)\}$ and redefine $s_{M+1} = \max\{s_M, T\}$. For admissible controls, $M(\omega) < \infty$ almost surely. Summing over all $0 \leq i \leq M$, we have

$$V_1(s_0, X_{s_0}, Y_{s_0}, Z_{s_0}) \geq E\left[V_1(s_{M+1}, X_{s_{M+1}}, Y_{s_{M+1}}, Z_{s_{M+1}})\right] = E\left[u(X_T + \sum_{i=1}^{N} (Y_{iT} - F_i)^+)\right]$$

In addition, the equality holds when $(\tau_i, \delta_{\tau_i})$ is chosen as the controls. The proof is completed.

Proof of Proposition 4.1:

Proof. The proof is similar to the proof of Proposition 2.2. Let $((s_i, \eta_{s_i}), \zeta_t), i \geq 1$ be an arbitrary set of admissible control, and $s_0 = t$. Also denote

$$L_2^\zeta \phi = \frac{1}{2} \sigma_2^2 y \frac{\partial^2 \phi}{\partial y^2} + \mu_2 y \frac{\partial \phi}{\partial y} + r x \frac{\partial \phi}{\partial x} + \frac{1}{2} \sigma_1^2 \zeta^2 \frac{\partial^2 \phi}{\partial x^2} + (\mu_1 - r) \zeta \frac{\partial \phi}{\partial x} + \rho \sigma_1 \sigma_2 \zeta y \frac{\partial^2 \phi}{\partial x \partial y}$$

Applying Ito’s lemma, we have

$$E\left[\phi(X_{s_j}, Y_{s_j}, s_j) - \phi(X_{s_{j+1}}, Y_{s_{j+1}}, s_{j+1})\right] = E\left[\int_{s_j}^{s_{j+1}} \left(-\frac{\partial \phi}{\partial t} - L_2^\zeta \phi\right)(X_{s}, Y_{s}, s)ds\right] \geq 0$$

where $j \geq 0$. Therefore we have

$$E\left[\phi(X_{s_j}, Y_{s_j}, s_j) - \phi(X_{s_{j+1}}, Y_{s_{j+1}}, s_{j+1})\right] \geq E\left[\phi(X_{s_{j+1}}, Y_{s_{j+1}}, s_{j+1}) - \phi(X_{s_{j+1}}, Y_{s_{j+1}}, s_{j+1})\right]$$

In addition, upon the intervention, we have

$$\phi(X_{s_{j+1}}, Y_{s_{j+1}}, s_{j+1}) = \phi(X_{s_{j+1}}, Y_{s_{j+1}}, s_{j+1})$$

Therefore

$$\phi(X_{s_{j+1}}, Y_{s_{j+1}}, s_{j+1}) \leq \phi(X_{s_{j+1}}, Y_{s_{j+1}}, s_{j+1})$$

and

$$E\left[\phi(X_{s_j}, Y_{s_j}, s_j) - \phi(X_{s_{j+1}}, Y_{s_{j+1}}, s_{j+1})\right] \geq 0$$
For a sample path \( \omega \), let \( M(\omega) = \max\{k \geq 1 : s_k(\omega) \leq T < s_{k+1}(\omega)\} \) and redefine \( s_{M+1} = \max\{s_M, T\} \). For admissible controls, \( M(\omega) < \infty \) almost surely. Summing over all \( 0 \leq i \leq M \), we have

\[
\phi(X_{s_0}, Y_{s_0}, s_0) \geq E\left[ \phi(X_{s_{M+1}}, Y_{s_{M+1}}, s_{M+1}) \right] = E\left[ u(X_T + (Y_T - F)^+) \right]
\]

In addition, the equality holds when \( ((\tau_i, \delta_i), \xi_t) \) is chosen as the controls. The proof is completed.

**Numerical solution procedure for the model with committed consumption**

Here we provide the numerical procedure used to solve our model with committed consumption. The algorithm below are presented for general utility function \( u(W) \), hence it applies to both CARA and CRRA case.

We first make the following change of independent variables

\[
w = \frac{x - Ce^{-r(T-t)} + y}{1 + x - Ce^{-r(T-t)} + y}, \quad b = \frac{y}{x - Ce^{-r(T-t)} + y}, \quad \pi = \frac{\xi}{x - Ce^{-r(T-t)}},
\]

and define \( U(w, b, t) = V_2(x, y, t) \). Then \( 0 \leq w \leq 1, 0 \leq b \leq 1, 0 \leq \pi \leq 1 \), and the HJB equation (16) is transformed into

\[
\begin{cases}
\min\{-\frac{\partial U}{\partial t} - L_0 U, \ U(w, b, t) - g(w, b, t)\} = 0, \quad (w, b) \in \Omega \\
U(0, b, t) = u(C), \ b \in [0, 1] \\
U(w, b, T) = u\left(\frac{w(1-b)}{1-w} + \left(\frac{bw}{1-w} - F\right)^+ + C\right), \ (w, b) \in \Omega
\end{cases}
\]

where \( \Omega = [0, 1] \times [0, 1] \) and

\[
g(w, b, t) = \begin{cases} -\infty, & 0 \leq w < \frac{F}{1+F}, \\
\sup_{\delta \in \Delta_0} U\left(\frac{w-F(1-w)}{1-F(1-w)}, \frac{wb+\delta(1-w)}{w-F(1-w)}, t\right), & \frac{F}{1+F} \leq w < 1.
\end{cases}
\]

\[\Delta_0 = \left[-\frac{wb}{1-w}, \frac{w(1-b)}{1-w} - F\right] \setminus \{0\}\]
and

\[
\mathcal{L}_0 U = a_1 \frac{\partial U}{\partial w} + a_2 \frac{\partial U}{\partial b} + a_3 \frac{\partial^2 U}{\partial w^2} + a_4 \frac{\partial^2 U}{\partial b^2} + a_5 \frac{\partial^2 U}{\partial w \partial b} + \max_{\pi \in [0,1]} \left\{ \pi \left[ b_1 \frac{\partial U}{\partial w} + b_2 \frac{\partial U}{\partial b} + b_3 \frac{\partial^2 U}{\partial w^2} + b_4 \frac{\partial^2 U}{\partial b^2} + b_5 \frac{\partial^2 U}{\partial w \partial b} \right] \right. \\
+ \left. \pi^2 \left[ c_1 \frac{\partial U}{\partial w} + c_2 \frac{\partial U}{\partial b} + c_3 \frac{\partial^2 U}{\partial w^2} + c_4 \frac{\partial^2 U}{\partial b^2} + c_5 \frac{\partial^2 U}{\partial w \partial b} \right] \right\}
\]

where the coefficients are

\[
\begin{align*}
a_1 &= w(1-w) \left[ r(1-b) + \mu_2 b - \sigma_2^2 b^2 w \right], \\
a_2 &= b(1-b) [\mu_2 - r - \sigma_2^2 b], \\
a_3 &= \frac{1}{2} \sigma_2^2 w^2 (1-w)^2 b^2, \\
a_4 &= \frac{1}{2} \sigma_2^2 b^2 (1-b)^2, \\
a_5 &= \sigma_2^2 w(1-w)b^2(1-b), \\
b_1 &= [(\mu_1 - r) - 2\rho \sigma_1 \sigma_2 w b]w(1-w)(1-b), \\
b_2 &= -(\mu_1 - r) + \rho \sigma_1 \sigma_2 (2b-1)]b(1-b), \\
b_3 &= \rho \sigma_1 \sigma_2 b(1-b)w^2(1-w)^2, \\
b_4 &= -\rho \sigma_1 \sigma_2 b^2(1-b)^2, \\
b_5 &= \rho \sigma_1 \sigma_2 b(1-b)(1-2b)w(1-w), \\
c_1 &= -\sigma_1^2 (1-b)^2 w^2(1-w), \\
c_2 &= \sigma_1^2 b(1-b)^2, \\
c_3 &= \frac{1}{2} \sigma_1^2 w^2(1-w)^2 (1-b)^2, \\
c_4 &= \frac{1}{2} \sigma_1^2 b^2 (1-b)^2, \\
c_5 &= -\sigma_1^2 w(1-w)b(1-b)^2.
\end{align*}
\]

Once again, we use the penalty method to numerically solve (20). The penalty equation is given by

\[
\begin{cases}
\frac{\partial U}{\partial t} + \mathcal{L}_0 U + \lambda (g(w,b,t) - U(w,b,t))^+ = 0, \quad (w,b) \in \Omega \\
U(0,b,t) = u(C), \quad b \in [0,1] \\
U(w,b,T) = u \left( \frac{w(1-b)}{1-w} + \left( \frac{bw}{1-w} - F \right)^+ + C \right), \quad (w,b) \in \Omega
\end{cases}
\]

(21)

where \( \lambda > 0 \) is a large penalty parameter. As \( \lambda \to \infty \), the solution to (21) converges to the solution to (20).
We discretize the time interval $[0, T]$ as $0 = t_0 < t_1 < \ldots < t_M = T$, and move backward in time. Given the function value $U^{n+1}$ at time $t_{n+1}$, we use the implicit finite difference scheme at time $t_n$

$$\frac{U^{n+1} - U^n}{t_{n+1} - t_n} + L_0 U^n + \lambda(g^n - U^n)^+ = 0$$

(22)

In order to deal with the nonlinear terms, we adopt the following iterative method, which combines Howard algorithm (cf. Howard (1960)) and non-smooth Newton iteration. The algorithm is as follows:

1. Set $l = 1$ and $U_0^n = U^{n+1}$;
2. Solve the linearized equation

$$\frac{U^{n+1} - U_l^n}{t_{n+1} - t_n} + L_0 U_l^n + \lambda(g^n_{l-1} - U_l^n) I_{\{g^n_{l-1} > U_l^n\}} = 0$$

(23)

where $U_l^n$ is the $l$th iterative value of $U^n$, and

$$L_0 U_l^n = a_1 \frac{\partial U_l^n}{\partial w} + a_2 \frac{\partial U_l^n}{\partial b} + a_3 \frac{\partial^2 U_l^n}{\partial w^2} + a_4 \frac{\partial^2 U_l^n}{\partial b^2} + a_5 \frac{\partial^2 U_l^n}{\partial w \partial b}$$

$$= \pi_{l-1} \left[ b_1 \frac{\partial U_{l-1}^n}{\partial w} + b_2 \frac{\partial U_{l-1}^n}{\partial b} + b_3 \frac{\partial^2 U_{l-1}^n}{\partial w^2} + b_4 \frac{\partial^2 U_{l-1}^n}{\partial b^2} + b_5 \frac{\partial^2 U_{l-1}^n}{\partial w \partial b} \right]$$

$$+ \pi_{l-1}^2 \left[ c_1 \frac{\partial U_{l-1}^n}{\partial w} + c_2 \frac{\partial U_{l-1}^n}{\partial b} + c_3 \frac{\partial^2 U_{l-1}^n}{\partial w^2} + c_4 \frac{\partial^2 U_{l-1}^n}{\partial b^2} + c_5 \frac{\partial^2 U_{l-1}^n}{\partial w \partial b} \right]$$

$$\pi_{l-1} = \arg \max_{\pi \in [0, 1]} \left\{ \pi \left[ b_1 \frac{\partial U_{l-1}^n}{\partial w} + b_2 \frac{\partial U_{l-1}^n}{\partial b} + b_3 \frac{\partial^2 U_{l-1}^n}{\partial w^2} + b_4 \frac{\partial^2 U_{l-1}^n}{\partial b^2} + b_5 \frac{\partial^2 U_{l-1}^n}{\partial w \partial b} \right] \right\}$$

where all the partial derivatives are discretized by suitable finite differences, and

$$g_{l-1}^n(w, b, t) = \begin{cases} -\infty, & 0 \leq w < \frac{F}{1+F}, \\ \sup_{\delta \in \Delta_0} U_{l-1}^n \left( \frac{w-F(1-w)}{1-F(1-w)}, \frac{w+b+\delta(1-w)}{w-F(1-w)}, t \right), & \frac{F}{1+F} \leq w < 1. \end{cases}$$

(3) Set $U^n = U_l^n$ if

$$\frac{|U_l^n - U_{l-1}^n|}{\max\{1, |U_{l-1}^n|\}} < \epsilon$$

where $\epsilon > 0$ is a pre-determined stopping threshold; Otherwise, set $l = l + 1$ and go to (2).
Table 8:
Baseline Parameter Values in the CRRA Model with Committed Consumption.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Baseline value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk free rate</td>
<td>$r$</td>
<td>0.01</td>
</tr>
<tr>
<td>Expected return of liquid stock</td>
<td>$\mu_1$</td>
<td>0.09</td>
</tr>
<tr>
<td>Volatility of liquid stock</td>
<td>$\sigma_1$</td>
<td>0.20</td>
</tr>
<tr>
<td>Expected return of illiquid stock</td>
<td>$\mu_2$</td>
<td>0.07</td>
</tr>
<tr>
<td>Volatility of illiquid stock</td>
<td>$\sigma_2$</td>
<td>0.15</td>
</tr>
<tr>
<td>Correlation between the returns</td>
<td>$\rho$</td>
<td>0</td>
</tr>
<tr>
<td>Committed consumption level</td>
<td>$C$</td>
<td>$100,000$</td>
</tr>
<tr>
<td>Fixed transaction cost</td>
<td>$F$</td>
<td>$5$</td>
</tr>
<tr>
<td>Investment horizon</td>
<td>$T$</td>
<td>5 years</td>
</tr>
<tr>
<td>Relative risk aversion</td>
<td>$\gamma$</td>
<td>3</td>
</tr>
</tbody>
</table>

Appendix B

Results for CRRA utility function

In this appendix, we show that our results remain valid with CRRA preferences. Specifically, we conduct the same analysis as in Section 5, except that we use a utility function of

$$u(W) = \frac{W^{1-\gamma}}{1-\gamma}, \quad W > 0$$

where $\gamma > 0$ and $\gamma \neq 1$ is the relative risk aversion coefficient. The same HJB equation, verification theorem and numerical solution procedure as those for the CARA case apply with the CARA utility function replaced with the CRRA utility function.

5.1 Optimal Trading Policy

Similar to the CARA case, we assume that the liquid stock has an expected return of 9% and volatility of 20%, the illiquid stock has an expected return of 7% and volatility of 15%, the correlation coefficient between the two stocks is 0.0, interest rate is 1%, fixed trading cost is $5, the relative risk aversion coefficient is 3, and the investment horizon is five years.

We report these baseline parameter values in Table 8.

We first plot the optimal trading policy at time 0 in Figure 5. When the portfolio weight (rather than dollar amount as in the CARA case) in the illiquid stock exceeds the selling boundary, the investor sell a lump sum amount of the stock so that the portfolio weight in the stock drops to the middle target ratio line. When the portfolio weight drops below
Figure 5: Optimal buying and selling boundary with CRRA utility, $t = 0$. Parameter values: $\mu_1 = 0.09$, $\sigma_1 = 0.2$, $\mu_2 = 0.07$, $\sigma_2 = 0.15$, $\rho = 0.0$, $r = 0.01$, $C = $100,000, $F = $5, $T = 5$, $\gamma = 3$. The buying boundary, the investor buys a lump sum amount so that the portfolio weight increases to the target ratio line. Trading direction is marked on the figure, and note that the total liquidation of the illiquid stock is possible when the disposable wealth is small. Intuitively, this is because when the disposable wealth is low, the investment in risky asset is small and thus the expected return dominates return risk and the investor optimally chooses to hold only the stock with the highest expected return, i.e., the liquid stock.

5.2 Disposition Effects

We now conduct the same analysis as before by computing the ratios PLR, PGR, and PGL using Monte Carlo simulations. We report the results in Table 9. Table 9 shows that even with a CRRA preference, the disposition effect exists across a large range of wealth levels. For example, at disposable wealth to the committed consumption ratio of 0.1, the PGR is equal to 0.574, while PLR is only 0.330. In addition, in all the sales, more than 86% are with a gain. The smallest disposition effect of $0.373 - 0.565 = -0.192$ is still statistically significant and occurs when the disposable wealth to the committed consumption ratio is 0.15. The lowest PGL is still as high as 82.6%. These results suggest that the assumption of a CARA preference in the main text is not critical for our results.
Table 9: Entire trading history
This table shows the results obtained from 100,000 simulated paths, with various initial disposable wealth $w_0$ (normalized by the committed consumption $C$). We show the results of mean and standard deviation (in the parenthesis). Parameter values: $\mu_1 = 0.09$, $\sigma_1 = 0.2$, $\mu_2 = 0.07$, $\sigma_2 = 0.15$, $\rho = 0.0$, $r = 0.01$, $C = $100,000, $F = $5, $T = 5$, $\gamma = 3$. The variable “DE” records the value of PLR-PGR.

<table>
<thead>
<tr>
<th>$w_0/C$</th>
<th>0.1</th>
<th>0.15</th>
<th>0.2</th>
<th>0.25</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.75</th>
<th>1</th>
<th>1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLR</td>
<td>0.330</td>
<td>0.373</td>
<td>0.214</td>
<td>0.156</td>
<td>0.141</td>
<td>0.119</td>
<td>0.134</td>
<td>0.149</td>
<td>0.152</td>
<td>0.276</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.003)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>PGR</td>
<td>0.574</td>
<td>0.565</td>
<td>0.717</td>
<td>0.710</td>
<td>0.693</td>
<td>0.685</td>
<td>0.670</td>
<td>0.653</td>
<td>0.643</td>
<td>0.609</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>DE</td>
<td>-0.244</td>
<td>-0.192</td>
<td>-0.504</td>
<td>-0.554</td>
<td>-0.552</td>
<td>-0.566</td>
<td>-0.536</td>
<td>-0.504</td>
<td>-0.491</td>
<td>-0.334</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.005)</td>
<td>(0.002)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>PGL</td>
<td>0.866</td>
<td>0.840</td>
<td>0.830</td>
<td>0.865</td>
<td>0.880</td>
<td>0.899</td>
<td>0.894</td>
<td>0.883</td>
<td>0.878</td>
<td>0.826</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.002)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
</tbody>
</table>

Table 10: Restricted to the sales in which the position in one stock is fully liquidated
This table shows the results obtained from 100,000 simulated paths, with various initial disposable wealth $w_0$ (normalized by the committed consumption $C$). We show the results of mean and standard deviation (in the parenthesis). Parameter values: $\mu_1 = 0.09$, $\sigma_1 = 0.2$, $\mu_2 = 0.07$, $\sigma_2 = 0.15$, $\rho = 0.0$, $r = 0.01$, $C = $100,000, $F = $5, $T = 5$, $\gamma = 3$. The variable “DE” records the value of PLR-PGR.

<table>
<thead>
<tr>
<th>$w_0/C$</th>
<th>0.1</th>
<th>0.15</th>
<th>0.2</th>
<th>0.25</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLR</td>
<td>1.000</td>
<td>0.703</td>
<td>0.217</td>
<td>0.165</td>
<td>0.182</td>
<td>0.219</td>
<td>0.269</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.004)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.003)</td>
<td>(0.008)</td>
<td>(0.020)</td>
</tr>
<tr>
<td>PGR</td>
<td>0.336</td>
<td>0.415</td>
<td>0.841</td>
<td>0.978</td>
<td>0.997</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>DE</td>
<td>0.664</td>
<td>0.288</td>
<td>-0.624</td>
<td>-0.813</td>
<td>-0.814</td>
<td>-0.781</td>
<td>-0.731</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.008)</td>
<td>(0.003)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>PGL</td>
<td>0.665</td>
<td>0.682</td>
<td>0.691</td>
<td>0.695</td>
<td>0.641</td>
<td>0.564</td>
<td>0.464</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.003)</td>
<td>(0.002)</td>
<td>(0.003)</td>
<td>(0.005)</td>
<td>(0.015)</td>
<td>(0.040)</td>
</tr>
</tbody>
</table>
To see if disposition effect can still appear in the subsample where an investor sells the entire position in a stock, we next restrict to the subsample paths along which the investor liquidates the entire illiquid stock position and report the corresponding results in Table 10. Table 10 shows that indeed our portfolio rebalancing model can generate the disposition effect when the disposable wealth to the committed consumption ratio is reasonably high. For example, when the disposable wealth to the committed consumption ratio is 0.2, we find the PGR is equal to 0.841, while PLR is only 0.217, implying a statistically significant disposition effect of -0.624. In addition, the fraction of sales that have gains is greater than 50% in most wealth levels, except when the disposable wealth to committed consumption ratio is equal to 0.5.